Advances in Engineering Software 48 (2012) 6–16

Contents lists available at SciVerse ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

MINLP optimization model for the nonlinear discrete time–cost trade-off problem

Uroš Klanšek *, Mirko Pšunder

University of Maribor, Faculty of Civil Engineering, Smetanova 17, 2000 Maribor, Slovenia

ARTICLE INFO

Article history:
Received 8 July 2008
Received in revised form 6 October 2009
Accepted 20 January 2012
Available online 18 February 2012

Keywords:
Project management
Scheduling
Time–cost trade-off problem
Cost optimization
Mixed-integer nonlinear programming (MINLP)

ABSTRACT

This paper presents the mixed-integer nonlinear programming (MINLP) optimization model for the nonlinear discrete time–cost trade-off problem (NDTCTP). The nonlinear total project cost objective function of the proposed MINLP optimization model is subjected to a rigorous system of generalized precedence relationship constraints between project activities, project duration constraints, logical constraints, and a budget constraint. By means of the proposed MINLP optimization model, one can obtain the minimum total project cost, the project schedule with the optimal discrete start times and the optimal discrete durations of the activities, as well as the optimal time–cost curves of the project. The proposed model yields the exact optimum solution of the NDTCTP. Solving the NDTCTP, using the proposed MINLP model, avoids the need for (piece-wise) linear approximation of the nonlinear expressions. The MINLP model handles the discrete variables explicitly and requires no rounding of the continuous solution into an integer solution. The applicability of the proposed optimization model is not limited to weakly NDTCTPs. A numerical example from the literature and an example of the project time–cost trade-off analysis are presented at the end of the paper in order to show the advantages of the proposed model.

1. Introduction

Cost effective project scheduling is an important issue in the planning process of a project. While the general scheduling of project activities is carried out before the submission of a tender, the executive project scheduling is performed before or during the realization of a project, Pšunder and Rebolj [1]. Execution of each project activity in normal duration requires employment of certain resources. Crashing an activity refers to the taking of additional resources at extra cost to reduce the duration of an activity below its normal value. The project time–cost trade-off optimization problem (TCTP) is concerned with crashing and scheduling a number of activities within a given project network in order to minimize the total cost of project execution. The TCTP is extensively discussed in project scheduling literature because of its practical relevance.

Since the development of the critical path method (CPM) in the late 1950s, the TCTP has received substantial attention from numerous researchers and has generated a considerable number of publications covering the areas of optimization methods and optimization models. Various different optimization models have been proposed to solve the TCTP using either approximate heuristic methods or exact mathematical programming methods.

Development of the optimization models for solving the TCTP using classical heuristic methods, such as genetic algorithms (GA) by Holland [2], simulated annealing (SA) by Kirkpatrick et al. [3], tabu search (TS) by Glover [4], neural networks (NNs) by Rumelhart et al. [5], ant colony optimization (ACO) by Dorigo et al. [6], and particle swarm optimization (PSO) by Kennedy and Eberhart [7], has been the subject of intensive research in project management. For instance, Feng et al. [8], Li et al. [9], Hegazy [10] and Leu and Yang [11] proposed the optimization models for solving the TCTP using the GA. Shtub et al. [12] and Azaron et al. [13] developed the SA model formulations. Gagnon et al. [14] proposed the TS optimization model to minimize the cost of the project. Adeli and Karim [15] have presented the NN model formulation for the cost optimal project scheduling. Xiong and Kuang [16] introduced an optimization model for solving the TCTP using the ACO algorithm. Recently, Yang [17] has proposed the PSO model formulation for the TCTP.

The mathematical programming methods can be separated into linear programming (LP), Dantzig [18]; nonlinear programming (NLP), Karush [19] and Kuhn and Tucker [20]; mixed-integer linear programming (MILP), Land and Doig [21]; and mixed-integer nonlinear programming (MINLP), Benders [22]. While the LP and the NLP perform the optimization of continuous variables, the MILP and the MINLP execute a simultaneous optimization of continuous and discrete variables. Kelley [23] and Fulkerson [24] were the pioneers in formulating the TCTP as the LP model. However, even the earliest studies in this field recognized the nonlinear nature of the TCTP. Therefore, Kapur [25], Deckro et al. [26] and Klanšek and

* Corresponding author. Address: University of Maribor, Faculty of Civil Engineering, Smetanova 17, PO Box 224, 2000 Maribor, Slovenia. Tel.: +386 2 22 94 378; fax: +386 2 25 24 179.

E-mail address: uros.klansek@uni-mb.si (U. Klanšek).

0965-9978/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.advengsoft.2012.01.006

Nomenclature

Indices
- e discrete solution alternative, $e \in E$
- i project activity, $i \in I$
- j succeeding project activity, $j \in J$

Constants
- BP total available project budget
- $DD_{i,e}$ alternative discrete solution for the activity duration
- $DS_{i,e}$ alternative discrete solution for the activity start time
- DT target project duration
- $FF_{i,j}$ 0–1 coefficient of the Finish-to-Finish precedence relationship
- $FS_{i,j}$ 0–1 coefficient of the Finish-to-Start precedence relationship
- $L_{i,j}$ lag/lead time between activities
- $SF_{i,j}$ 0–1 coefficient of the Start-to-Finish precedence relationship
- $SS_{i,j}$ 0–1 coefficient of the Start-to-Start precedence relationship

Variables
- B project award bonus
- C_i activity direct cost
- CI indirect project cost
- CT total project cost
- D_i activity duration
- DE amount of time the project is early
- DL amount of time the project is late
- DP actual project duration
- P project penalty cost
- S_i activity start time
- $yD_{i,e}$ binary 0–1 variable for selection of discrete solution for the activity duration
- $yS_{i,e}$ binary 0–1 variable for selection of discrete solution for the activity start time

Pšunder [27] have developed the NLP models for solving the nonlinear TCTP. Since the nature of project scheduling problems is also discrete, meaning, that the project schedule is usually determined by the discrete time units (e.g. working day), the discrete optimization must be applied to handle the discrete variables explicitly. Meyer and Shaffer [28] proposed one of the first MILP optimization models, where they addressed the discrete time–cost relationships of the TCTP. A few years later, similar MILP optimization models for the TCTP were developed by Crowston and Thompson [29] and Crowston [30]. Thus, the MILP optimization models have been mainly applied to solve the discrete TCTP. The recent progress in this field may be found in contributions presented by Achuthan and Hardjawidjaja [31], Sakellaropoulos and Chassiakos [32], Vanhoucke [33], Akkan et al. [34]. Since the MILP can handle only linear relations between the variables, the nonlinear terms of the optimization model are usually approximated with linear or piece-wise linear functions.

On the other hand, this paper presents the MINLP optimization model for solving the nonlinear discrete TCTP (NDTCTP). The nonlinear project cost objective function of the proposed MINLP optimization model is subjected to a rigorous system of generalized precedence relationship constraints between project activities, the project duration constraints, the logical constraints, and the budget constraint. By means of the proposed MINLP optimization model, one can obtain the minimum total project cost, the project schedule with the optimal discrete start times and the optimal discrete durations of the activities, as well as the optimal time–cost curves of the project. The proposed model yields the exact optimum solution of the NDTCTP. Solving the NDTCTP, using the proposed MINLP model, avoids the need for (piece-wise) linear approximation of the nonlinear expressions. The MINLP model handles the discrete variables explicitly and requires no rounding of the continuous solution into an integer solution. The applicability of the proposed optimization model is not limited to weakly NDTCTPs. A numerical example from the literature and an example of the project time–cost trade-off analysis are presented at the end of the paper in order to show the advantages of the proposed model.

2. MINLP optimization approach

The MINLP is a type of mathematical programming method which performs the discrete optimization of discrete parameters simultaneously with the continuous optimization of continuous parameters. Since the MINLP can handle nonlinear relations between the variables, it was selected to solve the NDTCTP of project scheduling. The general nonlinear continuous/discrete optimization problem may be formulated as a MINLP problem in the following form:

$$
\begin{aligned}
&\min z = \boldsymbol{c}^T\boldsymbol{y} + f(\boldsymbol{x})\\
&\text{s.t. } \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}\\
&\boldsymbol{g}(\boldsymbol{x}) \leqslant 0\\
&\boldsymbol{By} + \boldsymbol{Cx} \leqslant \boldsymbol{b}\\
&\boldsymbol{x} \in X = \left\{x \in R^n : \boldsymbol{x}^{LO} \leqslant \boldsymbol{x} \leqslant \boldsymbol{x}^{UP}\right\}\\
&\boldsymbol{y} \in Y = 0, 1^m
\end{aligned}
\qquad (\text{MINLP} - \text{G})
$$

where $\boldsymbol{x}$ is the vector of the continuous variables defined within compact set X and $\boldsymbol{y}$ is the vector of the binary 0–1 variables. While the continuous variables $\boldsymbol{x}$ can appear linearly or nonlinearly in the objective and constraints, the binary 0–1 variables $\boldsymbol{y}$ can only appear linearly. Functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ represent nonlinear functions involved in the objective function z, equality and inequality constraints, respectively. Finally, $\boldsymbol{By} + \boldsymbol{Cx} \leqslant \boldsymbol{b}$ denotes a subset of mixed linear equality and inequality constraints. All functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ must be continuous and differentiable.

With respect to the NDTCTP, the continuous variables define schedule parameters such as activity start times, activity durations, direct costs, and project duration. The binary 0–1 variables represent the potential selection of discrete solutions for continuous variables inside the project schedule superstructure. Equality and inequality constraints represent a rigorous system of generalized precedence relationship constraints, project duration constraints, and a budget constraint. The logical constraints that must be fulfilled for the discrete decisions and the schedule configurations, which are selected from within the superstructure, are defined by a subset of mixed linear equality and inequality constraints.

It is proposed that the MINLP optimization approach be carried out in three steps, see Klanšek et al. [35]. The first one includes the generation of a superstructure with different alternatives for discrete solutions of continuous variables, the second one involves the development of the MINLP optimization model formulation and the last one consists of a solution for the defined MINLP optimization problem.

Developing a suitable superstructure of discrete alternatives is an essential step in the MINLP optimization. This is because the