Hybrid LS–SA–PS methods for solving fuzzy non-linear programming problems

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ABSTRACT

The fuzzy optimization problem is one of the prominent topics in the broad area of artificial intelligence. It is applicable in the field of non-linear fuzzy programming. Its application as well as practical realization can been seen in all the real world problems. In this paper a large scale non-linear fuzzy programming problem was solved by hybrid optimization techniques like Line Search (LS), Simulated Annealing (SA) and Pattern Search (PS). An industrial production planning problem with a cubic objective function, eight decision variables and 29 constraints was solved successfully using the LS–SA–PS hybrid optimization techniques. The computational results for the objective function with respect to vagueness factor and level of satisfaction has been provided in the form of 2D and 3D plots. The outcome is very promising and strongly suggests that the hybrid LS–SA–PS algorithm is very efficient and productive in solving the large scale non-linear fuzzy programming problem.

1. Introduction

This paper discusses a real-world industrial problem for product-mix selection [1] involving eight variables and 21 constraints with fuzzy technological coefficients and thereafter, a formulation for an optimization approach to solve the problem. This problem occurs in production planning in which a decision maker plays a pivotal role in making decision under a fuzzy environment. The decision-maker should be aware of his level of satisfaction as well as degree of fuzziness while making product-mix decisions. Thus, a thorough analysis was performed on a modified *S*-curve membership function for the fuzziness patterns and fuzzy sensitivity solutions found from the various optimization methodologies in [1–11].

In this paper a hybrid optimization method is proposed to capture multiple non-dominated solutions in a single run of the algorithm. The obtained results have been compared with the well-known various hybrid evolutionary algorithms [4]. The combination of two or more hybrid methods to solve non-linear optimization problems has been shown to improve the results and to overcome drawbacks and weaknesses of each method separately.

In this paper, the line search method is focused on industrial production planning problems. The main advantage of this method is its ability to locate the near global optimal solutions for the fitness function with its strong criteria of global convergence. The line search method used the fmincon approach from the MATLAB computational toolbox. The Levenberg–Marquardt algorithm is a gradient-based method. It uses the sequential quadratic programming principles. In this method, the function solves a quadratic programming (QP) of the sub-problem at each iteration. A line search uses a merit function similar to that proposed by Powel [12]. The QP sub-problem uses an active set strategy as in [12]. A full description of this algorithm was found in [12].

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Constrained nonlinear Optimization Problems (COP) often take place in many practical applications such as construction planning, industrial process optimization, manufacturing optimization systems and so on [13]. These problems are challenging in terms of identifying feasible solutions when constraints are linear and the objective functions are nonlinear. Therefore, finding the location of the global optimum in the non-linear COP is more difficult as compared to linear bound-constrained global optimization problems. This research proposes a Hybrid Simulated Annealing method (HSA), for solving the general COP. HSA has features that address both feasibility and optimality issues and here, it is supported by a local search (LS) procedure, the gradient based method. In this research we developed a new hybrid method that combines the Simulated Annealing (SA), Pattern Search (PS) and Line Search (LS) – referred to as the hybrid LS–SA–PS method – in the context of fuzzy optimization of industrial production planning problems with non-linear objective functions.

The pattern search methods are class of direct search methods of nonlinear optimization. The original pattern search methods were introduced in late 1950s and early 1960s by Abramson [14] and remained popular due to their simplicity and the fact that they work well in the practice on various problems. Recently the nonlinear programming community renewed its interest on optimization, the fact that they are provably convergent [14].

The Pattern Search algorithm (PS) is a valuable one, but the application of non-smooth analysis techniques in [15] showed its limitations due to the finite choice of directions in [16]. The Mesh Adaptive Direct Search (MADS) removes the PS restriction to finitely many poll directions. We have long felt that this was the major impediment to stronger proofs of optimality for PS limit points (and better behavior), and a more satisfying optimality process for MADS in addition to opening new possibilities for handling the non-linear objective functions.

We list two attractive features of the pattern search algorithms:

- They can be extremely simple to specify and implement.
- Neither explicit calculations of derivatives nor anything like Taylor's series appear in the algorithms.

This makes these algorithms useful in situations where derivatives are not available and finite-difference derivatives are unreliable, such as when $f(x)$ is noisy.

These qualities made pattern search algorithms popular with users. Yet despite their seeming simplicity and heuristic nature and the fact that they have requirements to their derivatives of $f(x)$, the pattern search algorithms possess global convergence properties that are almost as strong as those of comparable line-search and trust-region algorithms. This surprising fact was explained by Lewis et al. [17].

These papers showed some further features of pattern search, which are used in their research work.

- They required only monotonic decreasing function $f(x)$. In fact, they do not require any numerical values of $f(x)$, but for information about improvement values of $f(x_+)$ on k-th step.
- If they are lucky, they need only one evaluation of $f(x)$ after an iteration. Once they can find an x_+ for which $f(x_+) < f(x_k)$, they can accept it and proceed. On the other hand, in the worst case they will look in quite a few directions ($2n$, for example) before they try shorter steps.
- The allowed steps are restricted in direction and length. i.e., the step direction must be parallel to the coordinate axes and the length of any step, which has the form $\Delta_0/2^N$ for some integer N.

The example also assumed that there is a great deal of flexibility in the pattern search algorithm, depending on how one specifies the pattern of points to be searched for the next iteration; see [17].

The pattern search algorithms are globally convergent, see [17] work since

(1) at each iteration, they look in enough directions to ensure that a suitably good descent direction will ultimately be considered;
(2) they possess a reasonable back-tracking strategy that avoids unnecessarily short steps;
(3) they otherwise avoid unsuitable steps by restricting the nature of the step allowed between successive iterates, rather than by placing requirements on the amount of decrease realized between successive iterates.

At the heart of the argument lies an unusual twist: Lewis et al. [17] relaxed the requirement of sufficient decrease and require only simple decrease ($f(x_{k+1}) < f(x_k)$), but Lewis et al. [17] imposed stronger conditions on the form the step s_k may take. Furthermore, this trade-off is more than just a theoretical innovation: in practice, it permits useful search strategies that are precluded by the condition of sufficient decrease.

The paper presents a new approach for obtaining the best optimal solution of the objective function and values of decision variables with minimal computational CPU time. This approach combines a line search heuristic with simulated annealing and pattern search. Computational experiments for these algorithms involve an industrial production planning problem and show simulation results. The remarkable performance of these techniques is revealed in this paper. The improvement in the computational efficiency of CPU timing has also been investigated in detail.

The LS has been used in the early stage to define an initial guess, then the hybrid SA–PS has been implemented in the final stage to finely tune the optimal results for the objective function.

The paper is organized as follows: Section 2 provides the problem statement; Section 3 presents a description of the proposed hybrid algorithm; the analysis and simulation results are included in Section 4, followed by conclusions and future research directions.