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Random fuzzy multi-objective linear programming: Optimization of possibilistic value at risk (pVaR)

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ABSTRACT

This paper considers multiobjective linear programming problems (MOLPP) where random fuzzy variables are contained in objective functions and constraints. A new decision making model optimizing possibilistic value at risk (pVaR) is proposed by incorporating the concept of value at risk (VaR) into possibility theory. It is shown that the original MOLPPs involving random fuzzy variables are transformed into deterministic problems. An interactive algorithm is presented to derive a satisficing solution for a decision maker (DM) from among a set of Pareto optimal solutions. Each Pareto optimal solution that is a candidate of the satisficing solution is exactly obtained by using convex programming techniques. A simple numerical example is provided to show the applicability of the proposed methodology to real-world problems with multiple objectives in uncertain environments.

1. Introduction

The real-world decision making situations can be often modeled in the framework of mathematical programming problems that involve multiple, noncommensurable, and conflicting objectives to be considered simultaneously (Kaiser & Messer, 2012; Miettinen, 1999; Steuer, 1985; Tamiz, 1996). Moreover, the objective functions and/or the constraints usually involve many parameters which are often uncertain. By extending stochastic programming (Birge & Louveaux, 2011; Dantzig, 1955; Infanger, 2011) to multiobjective programming, goal programming approaches (Contini, 1968; Stancu-Minasian, 1984) and interactive approaches (Goicoecha, Hansen, & Duckstein, 1982; Teghem, Dufrane, Thauvoye, & Kunsch, 1986; Urli & Nadeau, 2004) were presented. An overview of models and solution techniques for multiobjective stochastic programming problems were summarized in the context of Stancu-Minasian (1990). On the other hand, by considering the vague nature of the DM's judgments in multiobjective linear programming, fuzzy programming approaches have been studied (Kahraman, 2008; Lai & Hwang, 1994; Li & Hu, 2007, 2008; Lodwick & Kacprizyk, 2010; Rommelfanger, 1990; Tsuda & Saito, 2010; Verdegay, 2003; Werners, 1987; Zimmermann, 1978, 1985).

For the purpose of considering not only fuzziness but also randomness in decision making problems, fuzzy stochastic optimization has been studied (Hulsurkar, Biswal, & Sinha, 1997; Kato, Sakawa, Katagiri, & Wasada, 2004; Liu, 2004; Luhandjula, 1996, 2006; Luhandjula & Joubert, 2010; Luhandjula & Gupta, 1996; Wang, 2011) together with the development of mathematical basis on fuzzy probability and statistics (Buckley, 2006). In order to handle the situation that the realized values of random variables are fuzzy numbers, fuzzy random programming has been developed (Katagiri & Sakawa, 2011; Katagiri, Sakawa, & Ishii, 2005; Katagiri, Sakawa, Kato, & Ohsaki, 2005; Katagiri, Sakawa, Kato, & Nishizaki, 2004, 2008) by using the concept of fuzzy random variables (Kwakernaak, 1978; Puri & Ralescu, 1986; Wang & Qiao, 1993). These models handled decision making situations where the realized values of random variables in objective functions and/or constraints become fuzzy sets or fuzzy numbers.

However, we are faced with the situation where the mean of a random variable is estimated as a fuzzy set due to a lack of information. Such a parameter can be represented with a random fuzzy variable (Liu, 2002, 2004), instead of a fuzzy random variable.

Recently, a random fuzzy variable draws attention as a new tool for decision making problems under random fuzzy environments, such as random fuzzy programming (Katagiri, Ishii, & Sakawa, 2002; Liu, 2002), portfolio selection (Hasuike, Katagiri, & Ishii,

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2009), investment decision (Sakalli & Baykoc, 2010), project selection (Huang, 2007), facility location (Uno, Katagiri, & Kato, 2012) and cooperative bi-level programming (Katagiri, Niwa, Kubo, & Hasuike, 2010).

Under these circumstances, this article tackles a MOLPP with random fuzzy variables. Especially, we focus on the case where the uncertain parameters involved in problems are assumed to be estimated by Gaussian random variables with fuzzy mean, which can be represented with random fuzzy variables. In order to overcome the difficulty of obtaining an optimal solution of the problem due to the nonlinearity caused by simultaneous consideration of randomness and fuzziness, we shall propose a new decision making model in which the original random fuzzy problem can be transformed into an exactly-solvable deterministic MOLPP, using the ideas of possibility theory (Dubois & Prade, 2001; Zadeh, 1978) and value at risk (VaR) criterion (Jorion, 1996; Pritsker, 1997) (or fractile criterion Geoffrion, 1967; Kataoka, 1963) together with techniques of stochastic programming and possibilistic programming.

This paper is organized as follows. Section 2 introduces a definition of random fuzzy variables and provides a fuzzy set-based simple definition. Section 3 formulates a MOLPP with random fuzzy variables and proposes a decision making model optimizing possibilistic values at risk, called pVaR. Furthermore, we show the original random fuzzy MOLPP is transformed into a deterministic nonlinear MOLPP through the proposed model. The proposed model has an advantage that each Pareto optimal solution which is a candidate for a satisficing solution can be solved by convex programming techniques under realistic assumptions. In Section 5, we provide an interactive algorithm in order to obtain a satisficing solution for a DM from among a set of Pareto optimal solutions. After providing a simple numerical example in Section 4, We conclude this paper in Section 5.

2. Random fuzzy variables

When a random variable is used to express an uncertain parameter related to a stochastic factor of real systems, it is implicitly assumed that there exists a single random variable as a proper representation of the uncertain parameter. However, in some cases, experts may consider that it is suitable to employ a set of random variables, rather than a single one, in order to more precisely express the uncertain parameter. In this case, depending on the degree to which experts convince that each element (random variable) in the set is compatible with the uncertain parameter, it would be quite natural to assign the different values (different degrees of possibility) to the elements in the set. For handling such real-world decision making situations, a random fuzzy variable was introduced by Liu (2002) and explicitly defined (Liu, 2004) as a function from a possibility space to a collection of random variables.

In this section, we firstly introduce the Liu's original definition of random fuzzy variables, which is considered to be an extended concept of fuzzy variable (Nahmias, 1978). Next, we show a simple definition of random fuzzy variables (Katagiri, Kato, & Hasuike, 2012) as a natural extension of fuzzy sets.

2.1. Original definition of a random fuzzy variable

In preparation for introducing a random fuzzy variable, the definition of possibility space is given as follows:

Definition 1 (*Possibility space (Liu, 2004)*). Let Θ be a nonempty set, and $P(\Theta)$ be the power set of Θ. For each $A \in P(\Theta)$, there is a nonnegative number $Pos\{A\}$, called its possibility, such that

1. $Pos\{\emptyset\} = 0$, $Pos\{\Theta\} = 1$; and
2. $Pos\{\cup_k A_k\} = \sup_k Pos\{A_k\}$ for any arbitrary collection $\{A_k\}$ in $P(\Theta)$.

The triplet $(\Theta, P(\Theta), Pos)$ is called a possibility space, and the function Pos is referred to as a possibility measure.

Then, a random fuzzy variable is firstly defined by Liu (2004) as a function from a possibility space to a collection of random variables.

Definition 2 (*Random fuzzy variable (Liu, 2004)*). A random fuzzy variable is defined as a function from the possibility space $(\Theta, P(\Theta), Pos)$ to the set of random variables.

An example of random fuzzy variables are given by Liu (2004) as follows:

Example 1 (Liu, 2004). Assume that $\bar{\eta}_1, \bar{\eta}_2, \ldots, \bar{\eta}_m$ are random variables and $u_1, u_2, \ldots, u_m$ are real numbers in [0,1] such that $u_1 \vee u_2 \vee \cdots \vee u_m = 1$. Then $\bar{\tilde{\xi}}$ is a random fuzzy variable expressed as

$$\bar{\tilde{\xi}} = \begin{cases} \bar{\eta}_1 & \text{with possibility} \quad u_1, \\ \bar{\eta}_2 & \text{with possibility} \quad u_2, \\ & \cdots \\ \bar{\eta}_m & \text{with possibility} \quad u_m. \end{cases} \tag{1}$$

It should be noted here that $\bar{\tilde{\xi}}(i) = \bar{\eta}_i, \ i = 1, 2, \ldots, m$ are regarded as functions from a possibility space $(\Theta, P(\Theta), Pos)$ to a collection of random variables Γ if we define $\Theta = \{1, 2, \ldots, m\}$, $Pos\{i\} = u_i$, $i = 1, 2, \ldots, m$ and $\Gamma = \{\bar{\eta}_1, \bar{\eta}_2, \ldots, \bar{\eta}_m\}$.

Similar to the Nahmias's approach (Nahmias, 1978), the membership function of a random fuzzy variable is defined as follows:

Definition 3 (*Membership function of a random fuzzy variable (Liu, 2004)*). Let $\bar{\tilde{\xi}}$ be a random fuzzy variable on the possibility space $(\Theta, P(\Theta), Pos)$. Then its membership function is derived from the possibility measure Pos by

$$\mu(\bar{\eta}) = Pos\{\theta \in \Theta | \bar{\tilde{\xi}}(\theta) = \bar{\eta}\}, \quad \bar{\eta} \in \Gamma. \tag{2}$$

2.2. Fuzzy set-based definition of a random fuzzy variable

The original definition of random fuzzy variables (Liu, 2004) is based on the concept of fuzzy variables that were defined by Nahmias (1978). Considering that the imprecision represented by fuzzy variables can be also handled by fuzzy sets, Katagiri et al. (2012) introduce a simple definition of random fuzzy variables as an extended concept of fuzzy sets. It should be noted here that this definition does not contradict the original definition, and that it provides another way of viewing random fuzzy variables from a different direction through the concept of fuzzy sets, instead of fuzzy variables.

Definition 4 (*Random fuzzy variable (Katagiri et al., 2012)*). Let Γ be a collection of random variables. Then, a random fuzzy variable $\bar{\tilde{C}}$ is defined by its membership function

$$\mu_{\bar{\tilde{C}}} : \Gamma \to [0, 1]. \tag{3}$$

Remark 1. In Definition 4, the membership function $\mu_{\bar{\tilde{C}}}$ assigns each random variable $\bar{\gamma} \in \Gamma$ to a real number $\mu_{\bar{\tilde{C}}}(\bar{\gamma})$. If Γ is defined as $\mathbb{R}$, then (3) becomes equivalent to the membership function of an ordinary fuzzy set. In this sense, a random fuzzy variable can be regarded as an extended concept of fuzzy sets.