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Intensity-based premium evaluation for unemployment insurance products

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HIGHLIGHTS

- We provide a flexible intensity-based evaluation method for insurance premiums.
- We connect $\mathbb{F}$-doubly stochastic Markov chains to estimators of general Cox models.
- We give intensity estimates based on a German labor market data set.
- We compute unemployment insurance premiums via Monte Carlo simulations.

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ABSTRACT

We present a flexible premium determination method for insurance products, in particular, for unemployment insurance products. The price is determined with the real-world pricing formula and under the assumption that the employment–unemployment progress of an insured person follows an $\mathbb{F}$-doubly stochastic Markov chain. The stochastic intensity processes are estimated for the German labor market, using Cox's proportional hazards model with time-dependent covariates on a sample of integrated labor market biographies. The estimation procedure is based on a counting process framework with stochastic compensators, which we show to be naturally connected to the class of $\mathbb{F}$-doubly stochastic Markov chains. Based on the statistical analysis, the prices are computed using Monte Carlo simulations.

1. Introduction

The debt crisis in the Euro zone with Cyprus, Greece, Ireland, Portugal and Spain having applied for financial assistance of the European Stability Mechanism (ESM) or the European Financial Stability Facility (EFSF), respectively, constitutes an immense challenge for Europe and the rest of the world. One of the core structural problems of the affected countries is a troubled labor market with unemployment rates around 25%. Beside the financial drawbacks for the unemployed people, these high levels of unemployment rates burden the public unemployment insurance systems as well as the private insurance sector, which has started to introduce special products against unemployment. The demand on modern, well elaborated and tested mathematical models for premium determination and risk mitigation for these kind of insurance products, but also other insurance products in general, is high and an ongoing field in actuarial research; see e.g. Bacinello et al. (2009), Biagini and Schreiber (2013), Biagini and Widenmann (2012, 2013) or Møller (1998). In times, where all sectors of an economy are closely connected, one main issue in

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this context is to consider the insurance market as part of a hybrid market, consisting among others of stocks, equities, commodities, fixed income and insurance products, all influenced by micro- and macro-economic factors. Hence, the correlations and dependencies of models for (unemployment) insurance products and other sources of randomness in hybrid markets need to be investigated thoroughly.

In this context, the present paper aims to introduce a flexible premium determination framework for unemployment insurance products, particularly for so-called payment protection insurances (PPIs): given some underlying payment obligation of the insured person, e.g. a loan, the insurance company will take over the installments during an unemployment period. In this way, the financial challenges for the insured persons and the credit default risk for the creditor are both reduced at the same time.

Our pricing model is generally based on a two-state switching process with state space $\{1, 2\}$ ($1 \widehat{=}$ employed, $2 \widehat{=}$ unemployed), generated by two stochastic intensity processes. Generally speaking, the intensity of a transition from employment to unemployment at time t characterizes the conditional instantaneous probability at time t for an employee to become unemployed, given the currently available information. The intensity of a transition from unemployment to employment is interpreted analogously. In regard to the aforementioned dependencies of the model in hybrid markets, we assume the intensity processes to be driven by individual-related as well as macro- and micro-economic covariate processes; see Eq. (1) in Section 2.

An adequate class for the two-state switching process, which allows for stochastic intensity processes, is the class of $\mathbb{F}$-doubly stochastic Markov chains, introduced by Jakubowski and Niewęgłowski (2010). It extends the notion of classical Markov chains.

As a general pricing rule, we adopt the real-world pricing formula; see Platen and Heath (2007). A first model, using $\mathbb{F}$-doubly stochastic Markov chains and the benchmark approach for pricing PPIs, is proposed by Biagini and Widenmann (2012). However, the intensities there are assumed to be random, but not varying over time. In the present paper we extend this approach in order to address the more realistic case of stochastic intensity processes. However, in this case it is generally not possible to obtain an analytical expression for the insurance premium which will be here computed by using Monte Carlo simulations.

In order to calibrate the price for the unemployment insurance products to real data, we estimate the intensity processes using Cox's proportional hazards model; see Cox (1972, 1975) and Andersen et al. (1993). Our data set is provided by the "Institut für Arbeitsmarkt- und Berufsforschung" (IAB) and contains a sample of integrated labor market biographies, including the duration of employment and unemployment periods between 1975 and 2008 of more than 1.5 million German individuals as well as several useful socio-demographic covariates, such as age, nationality, educational level, regional details, etc. In order to reflect additional dependencies of the intensity processes of macro-economic factors, we also incorporate further covariates such as time series for MSCI-world returns and German unemployment rates.

An advantage of using Cox's proportional hazards model is the availability of adequate implementations; see for example the R-packages corresponding to de Wreede et al. (2010), Jackson (2011) or Aalen et al. (2004). A Bayesian approach is proposed by Kneib and Hennerfeind (2008) and implemented in the software BayesX. Here, a major difficulty is to adequately operationalize the data set with regard to the software packages.

Technically, the implemented estimators are based on the theory on multivariate counting processes and their compensators, where the counting process is assumed to count subsequent jumps of the same kind of an underlying multi-state switching process. Given the martingale property of the compensated counting process, estimators for the compensators can be derived. In the present paper we extend the existing theory for (classical) Markov chains[1] and show that the class of $\mathbb{F}$-doubly stochastic Markov chains is the natural candidate for the underlying multi-state switching processes corresponding to the multivariate counting processes with stochastic compensators of the form given by Cox's proportional hazards model. This relation can easily be extended to general multiplicative hazard models as given in Andersen et al. (1993).

In order to test the obtained estimation results, we apply conventional goodness-of-fit methods. The results generally show adequate performance of the estimated model parameters. Moreover, we introduce a further, non-standard method for testing the applicability of the obtained parameters with respect to prediction, by comparing actual and simulated jump times for selected paths from the data set. The results here show good predictive power, which implicates robustness of the Monte Carlo simulations to compute the premiums. A sensitivity analysis of the insurance premiums also confirms these findings.

Our approach, therefore, represents a flexible premium determination tool for unemployment insurance products since it takes into account various risk factors. Moreover, it can easily be adapted to model and estimate stochastic intensities and dependence structures in many other different applications of financial and actuarial practice as well as from other fields.

The rest of the paper is structured as follows. In Section 2 we introduce the form of the considered unemployment insurance products and derive the pricing formula. The connection between the multivariate counting processes of Cox's proportional hazards model and the class of $\mathbb{F}$-doubly stochastic Markov chains is established in Section 3. Additionally, the data set is described and estimation results for the intensity processes are presented. The Monte Carlo simulation procedure is explained in detail in Section 4 followed by a sensitivity analysis of the insurance premiums.

2. Unemployment insurance contracts

We now give a brief overview about the characteristics of unemployment insurance contracts. The product's basic idea is that the insurance company compensates to some extent the financial deficiencies, which an unemployed insured person is exposed to. We only consider contracts with deterministic, a priori fixed claim payments c_i which possibly take place at predefined payment dates $T_i,\ i = 1, \ldots, N$. Hence, the randomness of the claims is only due to their occurrence and not to their amount. As a practical example, one could think of PPI products against unemployment, which are linked to some payment obligation of an obligor to its creditor. The claim amount here is defined by the (a priori known) installments, which are paid at predefined payment dates.

In order to conclude the insurance contract, the insured person must be employed at least for a certain period before the beginning of the contract. We therefore assume that all insured persons are employed at $t = 0$ almost surely. The contract's exclusion clauses define three time periods, specifying whether the insured person is entitled to receive a claim payment or not:

– The *waiting period* starts with the beginning of the contract. If an insured person becomes unemployed at any time of this period, she is not entitled to receive any claim payments during this unemployment period.

[1] The classical time-inhomogeneous Markov chains generally have deterministic matrix-valued intensity functions of time. The corresponding counting processes also have deterministic compensators.