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A new algorithm to solve fully fuzzy linear programming problems using the MOLP problem

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ABSTRACT

Recently, two new algorithms have been proposed to solve a fully fuzzy linear programming (FFLP) problem by Lotfi et al. [F.H. Lotfi, T. Allahviranloo, M.A. Jondabeha, L. Alizadeh, Solving a fully fuzzy linear programming using lexicography method and fuzzy approximate solution, Appl. Math. Model. 33 (2009) 3151–3156] and Kumar et al. [A. Kumar, J. Kaur, P. Singh, A new method for solving fully fuzzy linear programming problems, Appl. Math. Model. 33 (2011) 817–823]. In this paper, based on a new lexicographic ordering on triangular fuzzy numbers, a novel algorithm is proposed to solve the FFLP problem by converting it to its equivalent a multi-objective linear programming (MOLP) problem and then it is solved by the lexicographic method. By a theorem, it is shown that the lexicographic optimal solution of MOLP problem can be considered as an optimal solution of the FFLP problem. Then, a simple example and two real problems, as two case studies, will be used to illustrate our algorithm and compare it with the existing methods.

1. Introduction

Nowadays, the decision of human is increasingly depend on information more than ever. But, most of information is not deterministic and in this situation human has a capability to make a rational decision based on this uncertainty. This is hard challenge for decision maker to design an intelligent system which make a decision the same as the human. So, it was led to propose a new concept of decision making in fuzzy environment by Bellman and Zadeh [1]. One of interesting concepts in fuzzy optimization problems is to deal with fuzzy linear programming (FLP) problems. Heretofore, a number of methods have been proposed to solve the FLP problems [2–11].

The FLP problem is said to be a fully fuzzy linear programming (FFLP) problem if all parameters and variables are considered as fuzzy numbers. Recently, two methods have been introduced to solve the FFLP problems by Lotfi et al. [12] and Kumar et al. [13]. In the first method [12], the parameters of FFLP problem have been approximated to the nearest symmetric triangular fuzzy numbers. After that, a fuzzy optimal approximation solution has been achieved by solving a multi-objective linear programming (MOLP) problem. The shortcoming exists of it is that the optimal solution of FFLP is not exact. So, it is not reliable solution for decision maker. In the second method [13], an exact optimal solution is achieved using a linear ranking function. In this method, the linear ranking function has been used to convert the fuzzy objective function to the crisp objective function. The shortcoming exists of it is that the fuzziness of objective function has been neglected by the linear ranking function.

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In this paper our motivation is to modify these two methods. First, the FFLP problem will be converted to a MOLP problem with three objective functions by use of a new lexicographic ordering on triangular fuzzy numbers. Then, the lexicographic method will be used to find a lexicographic optimal solution of MOLP problem. We prove that this lexicographic optimal solution can be considered as an exact optimal solution of FFLP problem. Finally, to show our advantages algorithm over existing methods [12,13], a simple example and two real world problems, namely the fuzzy transportation problem and the fuzzy investment problem, are used and compered our results with them.

This paper is organized as follows: In Section 2, some definitions and notations of the fuzzy numbers and also a new definition related to lexicographic ordering on triangular fuzzy numbers are presented. In Section 3, a new algorithm is proposed to solve the FFLP problem by converting to its equivalent MOLP problem. In Section 4, a simple example and two two case studies are used to illustrate the proposed algorithm. In Section 5, Lotfi's method, Kumar's method and the proposed algorithm will be compared to each other. Finally, conclusions are derived in Section 6.

2. Preliminaries

In this section, some definitions related to the fuzzy set theory, which will be used in the rest of paper, are given.

Definition 2.1. A fuzzy number is a fuzzy set like $u : \mathbb{R} \to I = [0, 1]$ which satisfies,

1. u is upper semi-continuous,
2. $u(x) = 0$ outside some interval $[c, d]$,
3. There are real numbers a, b such that $c \leqslant a \leqslant b \leqslant d$ and
   3.1 $u(x)$ is monotonic increasing on $[c, a]$,
   3.2 $u(x)$ is monotonic decreasing on $[b, d]$,
   3.3 $u(x) = 1, a \leqslant x \leqslant b$.

The set of all these fuzzy numbers is denoted by $F(\mathbb{R})$. An equivalent parametric form of that is presented as follows:

Definition 2.2. [14] A fuzzy number $\widetilde{u}$ in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$, $\overline{u}(r)$, $0 \leqslant r \leqslant 1$, which these functions satisfy the following requirements:

1. $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,
2. $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function,
3. $\underline{u}(r) \leqslant \overline{u}(r)$, $0 \leqslant r \leqslant 1$.

A popular fuzzy number is the trapezoidal fuzzy number $\widetilde{u} = (a, b, c, d)$ with interval defuzzifier $[b, c]$ and left fuzziness $(b - a)$ and right fuzziness $(d - c)$, where its membership function is given as follows:

$$\mu_{\widetilde{u}}(x) = \begin{cases} \frac{b-a}{x-a}, & \text{if } a \leqslant x \leqslant b, \\ 1, & \text{if } x \in [b, c], \\ \frac{d-c}{d-x}, & \text{if } c \leqslant x \leqslant d, \\ 0, & \text{Otherwise}. \end{cases}$$

Its parametric form is $\underline{u}(r) = a + (b - a)r, \overline{u}(r) = d + (c - d)r$ where $0 \leqslant r \leqslant 1$. In particular, $\widetilde{u} = (a, b, c, d)$ can also signify a triangular fuzzy number $\widetilde{u} = (a, b, d) = ((u)^l, (u)^c, (u)^u)$ if $b = c$. The set of all these triangular fuzzy numbers is denoted by $TF(\mathbb{R})$.

Definition 2.3. [14] A triangular fuzzy number $\widetilde{u} = (x_1, y_1, z_1)$ is said to be a non-negative triangular fuzzy number if and only if $x_1 \geqslant 0$. The set of all these triangular fuzzy numbers is denoted by $TF(\mathbb{R})^+$.

Definition 2.4. [14] Two triangular fuzzy numbers $\widetilde{u} = (x_1, y_1, z_1)$ and $\widetilde{v} = (x_2, y_2, z_2)$ are said to be equal, $\widetilde{u} = \widetilde{v}$, if and only if $x_1 = x_2, y_1 = y_2$ and $z_1 = z_2$.

Definition 2.5. [14] The arithmetic operations between two triangular fuzzy numbers are defined by the extension principle and can be equivalently represented as follows:

Let $\widetilde{u} = (x_1, y_1, z_1)$ and $\widetilde{v} = (x_2, y_2, z_2)$ be two triangular fuzzy numbers and $k \in \mathbb{R}$. Define.

(i) $k \geqslant 0, k\widetilde{u} = (kx_1, ky_1, kz_1)$,
(ii) $k \leqslant 0, k\widetilde{u} = (kz_1, ky_1, kx_1)$,
(iii) $\widetilde{u} \oplus \widetilde{v} = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$,
(iv) $\widetilde{u} \ominus \widetilde{v} = (x_1 - z_2, y_1 - y_2, z_1 - x_2)$,