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A dynamic programming approach for finding shortest chains in a fuzzy network

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ABSTRACT

Graph theory has numerous applications to problems in systems analysis, operations research, transportation, and economics. In many cases, however, some aspects of a graph-theoretic problem may be uncertain. For example, the vehicle travel time or vehicle capacity on a road network may not be known exactly. In such cases, it is natural to make use of fuzzy set theory to deal with the uncertainty. Here, we are concerned with finding shortest chains in a graph with fuzzy distance for every edge. We propose a dynamic programming approach to solve the fuzzy shortest chain problem using a suitable ranking method. By using MATLAB, two illustrative examples are worked out to demonstrate the proposed algorithm.

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1. Introduction

The shortest path problems are among the most important tasks of graph theory with many practical applications, e.g., in transportation, routing, and communication. They include such problems as finding the shortest path between two given vertices of a graph, finding the shortest paths from a given vertex to all other vertices, and finding the shortest paths between all pairs of vertices (the so-called shortest chain problem). While geographical distances can be stated deterministically, costs or times can fluctuate with traffic conditions, payload, and so on. In the past two cases, the reliability of deterministic value for the edge costs has been questioned. A typical alternative way is to allow for uncertainties in the edge costs by considering fuzzy numbers.

Here, we develop a fuzzy dynamic programming approach for shortest chain problems to find the shortest chain between every pair of nodes in an acyclic network with the following specifications. The arc lengths are be fuzzy numbers. For crisp arc lengths, there are several methods for finding the shortest path from a source node to a sink node based on dynamic programming, zero-one programming and also network flow theory. Some of these algorithms can be found in [26].

Numerous papers have been published for solving fuzzy graph problems [10,12,16,18,27].

Various approaches may be taken for solving fuzzy graph problems. Zadeh [29] shows that fuzzy graph may be viewed as a generalization of the calculi of a crisp graph. Blue et al. [2] give taxonomy of graph fuzziness that distinguishes five basic types combining fuzzy or crisp vertex sets with fuzzy or crisp edge sets and fuzzy weights and fuzzy connectivity. The fuzzy shortest path problem was first analyzed by Dubois and Prade [7]. They used Floyd's algorithm and Ford's algorithm [9,11] to treat the fuzzy shortest path problem. While the shortest path length can be obtained by their approach, but the corresponding path in the network may not exist. Klein [15] proposed a dynamic programming recursion-based fuzzy algorithm. Lin and Chen [17] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Another algorithm for this problem was presented by Okada and Gen [22,23], generalizing Djikstra's algorithm. In this algorithm, the weights of the arcs are considered to be interval numbers. Okada and Soper [21] proposed a fuzzy algorithm, which was based on multiple labeling methods to offer non-dominated paths to the decision maker. Blue et al. [2] presented an algorithm finding a cut value to limit the number of analyzed paths, and then applying a modified version of the k-shortest path (crisp) algorithm proposed by Eppstein [8]. Okada [20] introduced the concept of the degree of possibility of an arc being on the shortest path. Recently, Nayeem and Pal [19] proposed an algorithm based on the acceptance index introduced by Sengupta and Pal [25] and gave a single fuzzy shortest path or a guideline for choosing the best fuzzy shortest path according to the decision-maker's viewpoint. Chuang and Kung [5] proposed

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a fuzzy shortest path length procedure that could find a fuzzy shortest path length among all possible paths in a network. It is based on the idea that a crisp number is minimal if and only if any other number is larger than or equal to it.

It is patent that these articles present peculiarities and/or problems that warrant attention. Among the most important, there are the following three approaches: (1) they find arc lengths without an existing path; (2) they can determine a fuzzy solution set but do not provide decision-makers with any guidelines for choosing the best path [10]; (3) they find the shortest path between a source node and any other node.

We are thus motivated to propose an iterative algorithm for fuzzy shortest chain problem (FSCP). The algorithm is based on dynamic programming. Since the fuzzy min operator based on the extension principle leads to non-dominated solutions, we propose another approach to solve the FSCP using a suitable fuzzy ranking function.

The remainder of this paper is organized as follows. In Section 2, some elementary concepts and related operations of fuzzy set theory are reviewed. In Section 3, a procedure based on dynamic programming approach to solve the fuzzy shortest chain problem using a suitable ranking method proposed. In Section 4, two examples are given to demonstrate the proposed algorithm. Finally, conclusions are given in Section 5.

2. Definition and preliminaries

Here, some basic definitions of fuzzy sets and fuzzy numbers are reviewed from [1,4,14,22]. The basic definitions and notations will be used throughout the paper.

Definition 1. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval [0,1]. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ [4].

Definition 2. A fuzzy number $\tilde{A}$ is a fuzzy convex subset of the real line satisfying the following conditions:

(a) $\mu_{\tilde{A}}(x)$ is piecewise continuous;
(b) $\mu_{\tilde{A}}(x)$ is normalized, that is, there exists $m \in \Re$ with $\mu_{\tilde{A}}(m) = 1$, where m is called the mean value of $\tilde{A}$ [4].

Definition 3. A triangular fuzzy number $\tilde{a}$ can be defined by a triplet (a_1, a_2, a_3). Its conceptual schema and mathematical form are shown by the following equation:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 < x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 < x \le a_3 \\ 0, & a_3 < x. \end{cases} \quad (1)$$

A triangular fuzzy number $\tilde{a}$ in the universe of discourse X that conforms to this definition has been shown in Fig. 1.

Definition 4. A trapezoidal fuzzy number $\tilde{a}$ can be defined by a quadruplet (a_1, a_2, a_3, a_4). Its conceptual schema and mathematical form are shown by the following equation:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x < a_2 \\ 1, & a_2 \le x < a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 < x \le a_4 \\ 0, & a_4 > x. \end{cases} \quad (2)$$

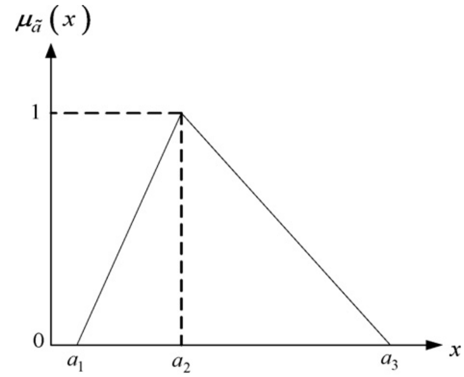

Fig. 1. A triangular fuzzy number $\tilde{a}$.

A trapezoidal fuzzy number $\tilde{a}$ in the universe of discourse X that conforms to this definition has been shown in Fig. 2.

Definition 5. Assuming that both $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ are triangular numbers, then the basic fuzzy operations are:

$$\tilde{a} \times \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \qquad \text{for multiplication}$$

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \qquad \text{for addition}$$

And if both $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ are trapezoidal numbers then the basic operations are:

$$\tilde{a} \times \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4) \qquad \text{for multiplication}$$

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \qquad \text{for addition}$$

Definition 6. The $\alpha - cut$, $\tilde{A}_\alpha$, and strong $\alpha - cut$, $\tilde{A}_{\alpha+}$, of the fuzzy set $\tilde{A}$ in the universe of discourse X is defined as:

$$\tilde{A}_\alpha = \{x | \mu_{\tilde{A}}(x) \ge \alpha, x \in X\}, \quad \text{where } \alpha \in [0,1] \quad (3)$$

$$\tilde{A}_{\alpha+} = \{x | \mu_{\tilde{A}}(x) > \alpha, x \in X\}, \quad \text{where } \alpha \in [0,1]. \quad (4)$$

The lower and upper points of any $\alpha - cut$, $\tilde{A}_\alpha$, are represented by $\inf\tilde{A}_\alpha$ and $\sup\tilde{A}_\alpha$, respectively, and we suppose that both are finite. For convenience, we denote $\inf\tilde{A}_\alpha$ by A_α^- and $\sup\tilde{A}_\alpha$ by A_α^+ (see Fig. 3) [7].

3. Fuzzy shortest chain problem

In this section a mathematical formulation of the fuzzy shortest chain problem and an algorithm for the fuzzy shortest chain problem are presented.

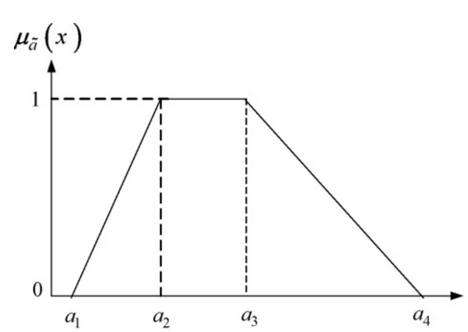

Fig. 2. A trapezoidal fuzzy number $\tilde{a}$.

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