Applied Mathematical Modelling 37 (2013) 4430–4439

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

# Optimal design of a sewer line using Linear Programming

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A R T I C L E   I N F O

*Article history:*
Received 14 March 2012
Received in revised form 5 August 2012
Accepted 11 September 2012
Available online 20 September 2012

*Keywords:*
Design
Optimisation
Roughness height
Sewer
Sewerage and wastewater collection

A B S T R A C T

Wastewater collection systems greatly contribute to the cost of the overall municipal sewerage system; a cost-effective design of the collection system will provide significant savings towards the cost of wastewater services. It is impossible to evaluate the full impact that each pipe size and slope would have on the overall cost of the collection system with intuitive designs. However, these solutions generally satisfy the design objectives within the given constraints. A survey of the literature indicates that various optimisation techniques are being applied for least-cost solutions. In general these approaches provide continuous pipe sizes, which are converted to closest commercial sizes for adoption, which would heavily dilute the optimal outcome. Search methods are also adopted to obtain cost-effective design solutions using directly commercial pipe sizes, which are computationally expensive. In the design of a sewerage system, a sewer line is a basic unit occurring repeatedly in the design-process and finally the combinations of these basic units formulate the complete sewer system. However, the branch sewer lines, main sewers, trunk sewers, pumping stations, treatment plant and outfall sewers are in general the main components of an urban wastewater collection, treatment and disposal systems. A method has been developed to optimise this basic unit using Linear Programming technique without transforming nonlinear objective function or constraint equations into linear functions and incorporating commercially available pipe sizes directly in the problem formulation. The current research area of optimal sewer system design is focusing equally on economic considerations and hydraulic feasibility and moving away from conventional design guidelines based on only self cleaning velocity concepts for node to node sewer link hydraulic design. This paper is a step forward in developing optimal design approaches of sewer systems.



## 1. Introduction

Wastewater from residential, commercial and industrial areas is collected and transported through the sewerage system to a sewage treatment plant, where it is treated to the specified standards before it is reused or disposed to receiving water. A sewerage system has a tree-like structure and is composed of various sewer lines which terminate at a junction that contains a larger sewer line. This larger sewer line further terminates at the junction of a still larger sewer line – the main sewer line – which eventually terminates at the wastewater treatment plant.

The hydraulic design of sewer system has not undergone any major change in the last 100 years; however a lot has been done in the construction and management of these systems. A typical system involves laying out a sewer network along

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http://dx.doi.org/10.1016/j.apm.2012.09.041

**Notations**

| | |
|---|---|
| $A$ | flow area |
| $c_e$ | unit excavation cost at ground level |
| $c_r$ | increase in unit excavation cost per unit depth |
| $c_s$ | sheeting and shoring cost coefficient |
| $D$ | link diameter |
| $d$ | nodal depth |
| $d_0$ | initial nodal depth |
| $d_f$ | outfall depth |
| $d_{\max}$ | maximum nodal depth |
| $d_n$ | terminal nodal depth |
| $g$ | gravitational acceleration |
| $h_{\min}$ | minimum cover depth |
| $k_D$ | diameter coefficient |
| $k_e$ | earthwork cost coefficient |
| $k_h$ | man hole cost coefficient |
| $k_m$ | pipe cost parameter |
| $L$ | link length |
| $m$ | pipe cost parameter |
| $n$ | number of links |
| $P$ | flow perimeter |
| $Q$ | link discharge |
| $Q_{\max}$ | carrying capacity of a link |
| $R$ | hydraulic radius |
| $S_o$ | invert slope of a link |
| $V$ | average fluid velocity |
| $V_{\max}$ | maximum fluid velocity |
| $V_s$ | scouring velocity |
| $V_{sc}$ | self-cleaning velocity |
| $w$ | trench width |
| $\varepsilon$ | average roughness height |
| $\gamma$ | Lagrange multiplier |
| $\nu$ | kinematic viscosity |
| $\eta$ | relative depth |

*Subscripts*

| | |
|---|---|
| $i$ | link or node; and |
| $j$ | index |

existing and proposed streets which terminates at the wastewater treatment plant – normally at the outskirts of urban boundary. Each sewer link is then designed as a separate element using some relationships governing the hydraulics of flow and a set of limiting constraints [1].

Camp [2] presented a method for the hydraulic design of sewer networks and highlighted the two main functions of the sewer systems: to carry the maximum discharge for which it is designed and to transport suspended solids. Since then many researchers [3–9] have contributed to the design of the sewer network and have applied various optimisation techniques. They have described heuristic methodologies for sewer design that could be adapted on microcomputers. Gupta et al. [10] used Powell's method of conjugate directions for depth–diameter optimisation of wastewater collection systems. Argaman et al. [11–14] applied dynamic programming for sewer systems design. Fisher et al. [15,1,16–18] used piecewise linearization to apply Linear Programming (LP) for estimating the pipe sizes and slopes. Swamee [19] developed a sewer line design method minimising nonlinear cost function and nonlinear constraints by iterative application of the Lagrange multiplier method. Genetic Algorithm (GA) is most popular and widely used search method. There are many examples of its application in sewer system design [20–24]. Hanghighi and Bakhshipour [25] highlighted that GAs slowly progress in a random-based framework and thus they are not computationally efficient compared to mathematical methods. As the number of variables and constraints increase the GAs become slow. They developed an adaptive genetic algorithm so that only feasible solutions are developed. The sewer pipe hydraulics estimates continuous pipe diameter which is rounded off to the first larger size in the commercial list for subsequent analysis. To overcome the slow progress of GAs some researchers linked this technique with other optimisation approaches. Cisty [26] hybridized the GA with LP and Haghighi et al. [27] hybridized GA with Integer LP for optimisation of water distribution system for improving efficiency. The integration of GA and LP could be