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Dynamic programming and Lagrange multipliers for active relaxation of resources in nonlinear non-equilibrium systems

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Abstract

In power production problems maximum power and minimum entropy production and inherently connected by the Gouy–Stodola law. In this paper various mathematical tools are applied in dynamic optimization of power-maximizing paths, with special attention paid to nonlinear systems. Maximum power and/or minimum entropy production are governed by Hamilton–Jacobi–Bellman (HJB) equations which describe the value function of the problem and associated controls. Yet, in many cases optimal relaxation curve is non-exponential, governing HJB equations do not admit classical solutions and one has to work with viscosity solutions. Systems with nonlinear kinetics (e.g. radiation engines) are particularly difficult, thus, discrete counterparts of continuous HJB equations and numerical approaches are recommended. Discrete algorithms of dynamic programming (DP), which lead to power limits and associated availabilities, are effective. We consider convergence of discrete algorithms to viscosity solutions of HJB equations, discrete approximations, and the role of Lagrange multiplier λ associated with the duration constraint. In analytical discrete schemes, the Legendre transformation is a significant tool leading to original work function. We also describe numerical algorithms of dynamic programming and consider dimensionality reduction in these algorithms. Indications showing the method potential for other systems, in particular chemical energy systems, are given.

Keywords: Resources; Power generation; Radiation; Dynamic programming

1. Introduction

In this paper we consider analytical and computational aspects of energy limits in dynamical systems propelled by nonlinear fluids that are restricted in their amount or flow, and, as such, play role of resources. In practical processes of engineering and technology a resource is a useful, valuable substance of a limited amount or flow. Value of the resource can be quantified thermodynamically by specifying its exergy, a

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Nomenclature

a	temperature power exponent in kinetic equation
c	specific heat ($\mathrm{J\,g^{-1}\,K^{-1}}$, $\mathrm{J\,m^{-3}\,K^{-1}}$, $\mathrm{J\,mol^{-1}\,K^{-1}}$)
D^n, $\widetilde{D}^n$	generalized profit and gauge profit at stage n
G	gauge function
$\dot{G}$	resource flux ($\mathrm{g\,s^{-1}}$, $\mathrm{mol\,s^{-1}}$)
$\mathbf{f}$	rate vector with components $f_1 \ldots f_k \ldots f_s$
f_0	intensity of generalized profit
H	Hamiltonian function
l_0	Lagrangian, intensity of generalized cost
R	minimum performance function (J, or $\mathrm{J\,mol^{-1}}$)
S	entropy ($\mathrm{J\,K^{-1}}$)
T	variable temperature of resource fluid (K)
T^n	temperature after stage n (K)
T^{e}	constant temperature of environment (K)
T'	Carnot temperature control (K)
$\dot{T}$	u rate of controlling of T in time τ (K)
t	time (s)
$\mathbf{u}$	control vector
u	temperature rate control, $\mathrm{d}T/\mathrm{d}\tau$ (K)
V	maximum performance function (J, or $\mathrm{J\,mol^{-1}}$)
W and $\dot{W}$	work and power (J, $\mathrm{J\,s^{-1}}$)
$\mathbf{x}$	state vector
$\tilde{x}$	enlarged state vector including time
z_k	adjoint variable

Greek symbols

β	coefficient, frequence constant ($\mathrm{s^{-1}}$)
λ	Lagrange multiplier, time adjoint
η	first-law efficiency (–)
θ	time interval (s,–)
Φ	factor of machine irreversibility (–)
ξ	intensity factor (–)
τ	nondimensional time or number of heat transfer units (x/H_{TU}) (–)

Subscripts

k	kth state variable
m	molar flow
1, 2	first and second fluid
*	modified cost or profit

Superscripts

e	environment
i	initial state
n-	stage number
f	initial state
$'$	modified quantity