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Interactive fuzzy programming for fuzzy random two-level linear programming problems through probability maximization with possibility

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A R T I C L E I N F O

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A B S T R A C T

This paper considers two-level linear programming problems involving fuzzy random variables under cooperative behavior of the decision makers. Through the introduction of fuzzy goals together with possibility measures, the formulated fuzzy random two-level linear programming problem is transformed into the problem to maximize the satisfaction degree for each fuzzy goal. By adopting probability maximization, the transformed stochastic two-level programming problem can be reduced to a deterministic one. Interactive fuzzy programming to derive a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented. An illustrative numerical example demonstrates the feasibility and efficiency of the proposed method.

1. Introduction

In actual decision making situations, we must often make a decision on the basis of vague information or uncertain data. For such decision making problems involving uncertainty, there exist two typical approaches: probability-theoretic approach (Charnes & Cooper, 1963; Stancu-Minasian, 1984, 1990; Birge & Louveaux, 1997; Kall & Mayer, 2011) and fuzzy-theoretic one (Zimmermann, 1978, 1987; Sakawa, 1993). However, in practice, decision makers are faced with the situations where both fuzziness and randomness exist. In the case where some expert estimates coefficients of objective functions or constraints with uncertainty, they do not always represent the coefficients as random variables or fuzzy sets but as the values including both fuzziness and randomness. In such a case, it is important to realize that simultaneous considerations of both fuzziness and randomness would be required. A fuzzy random variable (Kwakernaak, 1978) is one of the mathematical concepts dealing with fuzziness and randomness simultaneously. Studies on linear programming problems with fuzzy random variable coefficients, called fuzzy random linear programming problems, were initiated by Wang and Qiao (1993), Qiao, Zhang, and Wang (1994) as seeking the probability distribution of the optimal solution and optimal value. A brief survey of major fuzzy stochastic programming models was found in the paper by Luhandjula (2006).

On the other hand, we often encounter situations where there are two or more decision makers in an organization with a hierarchical structure, and they make decisions in turn or at the same time so as to optimize their objective functions. In particular, consider a case where there are two decision makers; one of the decision makers first makes a decision, and then the other who knows the decision of the opponent makes a decision. Such a situation is formulated as a two-level programming problem (Sakawa & Nishizaki, 2009). In the context of two-level programming, the decision maker at the upper level first specifies a strategy, and then the decision maker at the lower level specifies a strategy so as to optimize the objective with full knowledge of the action of the decision maker at the upper level. In conventional multi-level mathematical programming models employing the solution concept of Stackelberg equilibrium, it is assumed that there is no communication among decision makers, or they do not make any binding agreement even if there exists such communication (Bialas & Karwan, 1984; Shimizu, Ishizuka, & Bard, 1997; Simaan & Cruz, 1973). Compared with this, for decision making problems in such as decentralized large firms with divisional independence, it is quite natural to suppose that there exists communication and some cooperative relationship among the decision makers (Sakawa & Nishizaki, 2009).

For two-level linear programming problems or multi-level ones such that decisions of decision makers in all levels are sequential and all of the decision makers essentially cooperate with each other, Lai (1996) and Shih, Lai, and Lee (1996) proposed fuzzy interactive approaches. In their methods, the decision makers identify membership functions of the fuzzy goals for their objective functions, and in particular, the decision maker at the upper level also specifies those of the fuzzy goals for the decision variables. The decision maker at the lower level solves a fuzzy programming problem with a constraint with respect to a satisfactory degree of the decision maker at the upper level. Unfortunately, there is a

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possibility that their method leads a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and those of the decision variables. In order to overcome the problem in their methods, by eliminating the fuzzy goals for the decision variables, Sakawa et al. have proposed interactive fuzzy programming for two-level or multi-level linear programming problems to obtain a satisfactory solution for decision makers (Sakawa, Nishizaki, & Uemura, 1998, 2000a). Extensions to two-level linear fractional programming problems (Sakawa, Nishizaki, & Uemura, 2001), decentralized two-level linear programming problems (Sakawa & Nishizaki, 2002a; Sakawa, Nishizaki, & Uemura, 2002), two-level linear fractional programming problems with fuzzy parameters (Sakawa, Nishizaki, & Uemura, 2000b), and two-level nonconvex programming problems with fuzzy parameters (Sakawa & Nishizaki, 2002b) were provided. Further extensions to two-level linear programming problems with random variables, called stochastic two-level linear programming problems (Sakawa & Katagiri, 2010; Sakawa & Kato, 2009), two-level integer programming problems (Sakawa, Katagiri, & Matsui, 2010), and two-level linear programming problems involving fuzzy random variables, called fuzzy random two-level programming problems (Sakawa, Katagiri, & Matsui, 2011, 2011), have also been considered. It should be noted here that fuzzy random variables (Kwakernaak, 1978; Puri & Ralescu, 1986; Wang & Qiao, 1993) are considered to be random variables whose realized values are not real values but fuzzy numbers or fuzzy sets. A recent survey paper of Sakawa and Nishizaki (2012) is devoted to reviewing and classifying the numerous major papers in the area of so-called cooperative multi-level programming.

Under these circumstances, in this paper, assuming cooperative behavior of the decision makers, we consider solution methods for decision making problems in hierarchical organizations under fuzzy random environments. To deal with the formulated two-level linear programming problems involving fuzzy random variables, possibility measures are introduced and a stochastic two-level linear programming problem is defined for guaranteeing the degree of realization of the problem. Taking into account vagueness of judgments of decision makers, fuzzy goals are introduced and the stochastic two-level linear programming problem is transformed into the problem to maximize the satisfaction degree for each fuzzy goal. Following probability maximization, the transformed stochastic two-level programming problem can be reduced to a deterministic one. Interactive fuzzy programming to obtain a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented. It is shown that all of the problems to be solved in the proposed interactive fuzzy programming can be easily solved by the simplex method or the combined use of the bisection method and the simplex method.

2. Fuzzy random two-level linear programming problem

Fuzzy random variables, first introduced by Kwakernaak (1978), have been defined in various ways (Kwakernaak, 1978; Puri & Ralescu, 1986; Kruse & Meyer, 1987; Liu & Liu, 2003). For example, as a special case of fuzzy random variables given by Kwakernaak (1978), Kruse and Meyer (1987) defined a fuzzy random variable as follows.

Definition 1 (*Fuzzy random variable*). Let (Ω, B, P) be a probability space, $F(\mathcal{R})$ the set of fuzzy numbers with compact supports and X a measurable mapping $\Omega \to F(\mathcal{R})$. Then X is a fuzzy random variable if and only if given $\omega \in \Omega, X_\alpha(\omega)$ is a random interval for any $\alpha \in (0,1]$, where $X_\alpha(\omega)$ is an α-level set of the fuzzy set $X(\omega)$.

Although there exist some minor differences in several definitions of fuzzy random variables, fuzzy random variables are considered to be random variables whose observed values are fuzzy sets.

In this paper, we deal with two-level linear programming problems involving fuzzy random variable coefficients in objective functions formulated as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{11}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{12}\boldsymbol{x}_2 \\ \underset{\text{for DM2}}{\text{minimize}} & z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{21}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{22}\boldsymbol{x}_2 \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leqslant \boldsymbol{b} \\ & \boldsymbol{x}_1 \geqslant \boldsymbol{0},\ \boldsymbol{x}_2 \geqslant \boldsymbol{0} \end{array}\right\} \quad (1)$$

where $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the DM at the upper level (DM1), $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the DM at the lower level (DM2), $z_1(\boldsymbol{x}_1,\boldsymbol{x}_2)$ is the objective function for DM1 and $z_2(\boldsymbol{x}_1,\boldsymbol{x}_2)$ is the objective function for DM2.

It should be emphasized here that randomness and fuzziness of the coefficients are denoted by the "dash above" and "wave above" i.e., "-" and "∼", respectively. In this formulation, $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the DM at the upper level (DM1), $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the DM at the lower level (DM2), $z_1(\boldsymbol{x}_1,\boldsymbol{x}_2)$ is the objective function for DM1 and $z_2(\boldsymbol{x}_1,\boldsymbol{x}_2)$ is the objective function for DM2. In (1), $\tilde{\bar{\boldsymbol{C}}}_{lj},\ l = 1,2,\ j = 1,2$ are vectors whose elements $\tilde{\bar{C}}_{ljk},\ k = 1,2,\ldots,n_j$ are fuzzy random variables characterized by the following membership function:

$$\mu_{\tilde{\bar{C}}_{ljk}}(\tau) = \begin{cases} L\left(\frac{\bar{d}_{ljk}-\tau}{\beta_{ljk}}\right), & \text{if } \tau \leqslant \bar{d}_{ljk} \\ R\left(\frac{\tau-\bar{d}_{ljk}}{\gamma_{ljk}}\right), & \text{otherwise} \end{cases}$$

where L:$[0,\infty) \to [0,1]$ is a monotone decreasing function defined as $L(t) = \max\{0, \lambda(t)\}$ and R: $[0,\infty) \to [0,1]$ is also a monotone decreasing function defined as $R(t) = \max\{0, \rho(t)\}$. Here, $\lambda(t)$ and $\rho(t)$ are monotone decreasing functions which satisfy $\lambda(0) = 1$ and $\rho(0) = 1$, respectively. Furthermore, parameters $\bar{d}_{ljk}, \beta_{ljk}$ and γ_{ljk} represent a mean value, the left spread and the right spread of $\tilde{\bar{C}}_{ljk}$, respectively. In this paper, parameters $\bar{d}_{ljk}$ are random variables defined as $\bar{d}_{ljk} = d^1_{ljk} + \bar{t}_l d^2_{ljk}$, using random variables $\bar{t}_l$ with mean M_l, l = 1,2. This definition of random variables is one of the simplest randomization modeling of coefficients using dilation and translation of random variables, as discussed by Stancu-Minasian (1990).

Fig. 1 illustrates an example of the membership function of a fuzzy random variable $\tilde{\bar{C}}_{ljk}$.

Fuzzy random two-level linear programming problems formulated as (1) are often seen in actual decision making situations. For example, consider a supply chain planning (Roghanian, Sadjadi,

htb

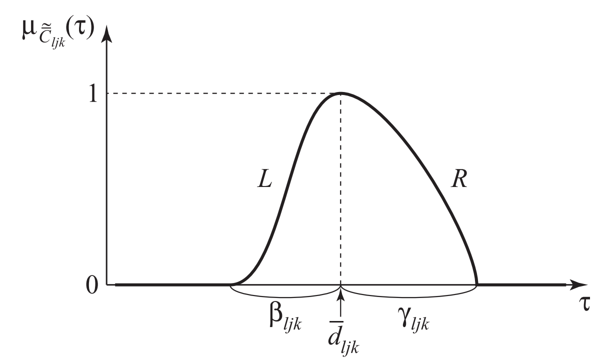

Fig. 1. An example of the membership function $\mu_{\tilde{\bar{C}}_{ljk}}(\cdot)$ of a fuzzy random variable $\tilde{\bar{C}}_{ljk}$.