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Solving fuzzy multi-objective linear programming problems using deviation degree measures and weighted max–min method

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ABSTRACT

This paper proposes a method for solving fuzzy multi-objective linear programming (FMOLP) problems where all the coefficients are triangular fuzzy numbers and all the constraints are fuzzy equality or inequality. Using the deviation degree measures and weighted max–min method, the FMOLP problem is transformed into crisp linear programming (CLP) problem. If decision makers fix the values of deviation degrees of two side fuzzy numbers in each constraint, then the δ-pareto-optimal solution of the FMOLP problems can be obtained by solving the CLP problem. The bigger the values of the deviation degrees are, the better the objectives function values will be. So we also propose an algorithm to find a balance-pareto-optimal solution between two goals in conflict: to improve the objectives function values and to decrease the values of the deviation degrees. Finally, to illustrate our method, we solve a numerical example.

1. Introduction

Multi-objective linear programming (MOLP) has important applications in many areas of engineering and management. In applications, one of the major works faced by experts and decision makers (DM) is to determine the values of parameters in MOLP model. Since real world problems are very complex, experts and DM frequently do not precisely know the values of those parameters. Therefore, it may be more appropriate to consider the knowledge of experts or DM about the parameters as fuzzy data. The fuzzy multi-objective linear programming (FMOLP) problems with fuzzy parameters would be viewed as a more realistic version than the conventional one [1,2].

Various kinds of FMOLP models have been proposed and the methods for solving them have also been developed in literatures, for example, Wu et al. [2], Deep et al. [3], Hu et al. [4]. The classification of these FMOLP models and their solution methods can be found in [5]. This paper considers the FMOLP problems where all the parameters in both objective functions and constraints are fuzzy numbers and all the constraints are fuzzy equality or inequality. The aim of this paper is to introduce a new method for solving this type of FMOLP problems using deviation degree measures and weighted max–min method.

In the proposed method we first transform the FMOLP problem into crisp linear programming (CLP) problem and then solve the CLP problem to obtain the pareto-optimal solution of the FMOLP problem. In the process of solving it, we should answer two key questions:

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(1) How to transform fuzzy constraints/objectives into crisp ones.

The methods appeared in the literature for transforming fuzzy constraints/objectives into crisp ones are mainly the methods based on ranking fuzzy numbers. Ranking of fuzzy numbers is not a simple process. Unlike real numbers, fuzzy numbers have no natural order [6]. There are two main approaches to ranking fuzzy numbers [7,8]. One of them consists in the definition of a ranking function mapping each fuzzy number into the real line. For example, Yao and Wu [9] used signed distance to define ranking of fuzzy numbers. Chen and Chen [10] presented a method to evaluate the ranking order between generalized fuzzy numbers based on center of gravity points and standard deviations of generalized fuzzy numbers. The other approach ranks fuzzy numbers by means of a fuzzy relationship. It allows the DM to present his/her preferences. For example, Jiménez [11] has proposed a ranking method of fuzzy numbers, which allows the DM to handle with different degrees of satisfaction of constraints.

In the selection of the ranking method, the shape of the fuzzy numbers, the obtained relation from the ranking method and the ease of computation of the ranking method can be the major factors to be considered [12]. Based on these considerations, this paper proposes a new ranking method of fuzzy numbers, which use deviation degree measures to rank the fuzzy objective values and to deal with the equality or inequality relation on constraints.

(2) How to aggregate multiple objectives problem into a single objective one.

Most of the methods available in literature aggregate multiple objectives problem into a single objective one by using some real-valued functions and utility functions [13]. The real-valued functions may take different forms, such as weighted sum [14], max–min or weighted max–min [15–19], the product form [19]. The utility functions methods are based on the DM's preference. This preference is translated to mathematical expression by using the utility function [13,20]. Different function forms may have different advantages and disadvantages. For instance, the weighted sum approach can't guarantee that the achievement levels of fuzzy goals are consistent with desirable relative weights or the DM's expectations [18]. The solution with product operator is always Pareto optimal [21], but due to its nonlinearity, it has not found much favor with the users [3].

In this paper, since max–min operator approach possesses some good properties [19], and the weighted max–min model finds an optimal solution within the feasible area such that the ratio of the achieved levels is as close to the ratio of the weights as possible [17], we choose weighted max–min approach to aggregate multiple objectives problem into a single objective one. A main work of this approach is to determine the weights of the deviation degrees in each objective. In the literatures [22,23], there are several methods for determining the weights of each objective. Abd El-Wahed [22] has proposed a method which can obtain the optimal differential weights of the objective functions by solving an auxiliary linear programming problem. Based on it, we use the *Technique for Order Preference by Similarity to Ideal Solution* (TOPSIS) method to determine the weights of the deviation degrees in each objective.

Therefore, using the deviation degree measures and weighted max–min method, we can transform the FMOLP problem into CLP problem, and then solve it to obtain the pareto-optimal solution of the FMOLP problems. Since the concept of the deviation degree is used to compare or rank fuzzy numbers of two sides in constrains, the solution of the FMOLP model will be influenced by the deviation degree. Moreover, the bigger the values of the deviation degrees fixed by DM are, the better the objectives function values will be. So, the DM have to find a balance-pareto-optimal solution between two objectives in conflict: to improve the objectives function values and to decrease the deviation degree of two side fuzzy numbers in constraints.

This paper is organized as follows: In Section 2 the basic definition of distance and deviation measure between two triangular fuzzy numbers is reviewed. In Section 3 a FMOLP problem with fuzzy parameters is discussed and the CLP model to solve the FMOLP problems is proposed using deviation degree measures and weighted max–min method. In Section 4 the methods for determining fuzzy aspiration level and the weights of the deviation degree in each objective are presented. In Section 5 the procedures for solving the pareto-optimal solution of the FMOLP problems are described. In Section 6 a numerical example is given. Conclusions are discussed in Section 7.

2. Deviation degrees of two triangular fuzzy numbers

There are many possible forms for a membership function: linear, exponential, hyperbolic, hyperbolic inverse, piecewise linear, etc. Díaz-Madroñero et al.[24], Peidro and Vasant [25] and Vasant et al.[26] had given a comparisons of major types of membership functions. In this paper, the triangular fuzzy numbers are considered because this form of fuzzy numbers is very simple and popular. Moreover, we can express and estimate many other types of fuzzy numbers with triangular fuzzy number [27]. The triangular fuzzy numbers is defined as follows:

Definition 2.1 (*[28,29]*). A fuzzy number $\widetilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ is said to be a triangular fuzzy number if its membership function is given by