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Mehar's method for solving fully fuzzy linear programming problems with *L-R* fuzzy parameters

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ABSTRACT

To the best of our knowledge, there is no method in literature for solving such fully fuzzy linear programming (FLP) problems in which some or all the parameters are represented by unrestricted *L-R* flat fuzzy numbers. Also, to propose such a method, there is need to find the product of unrestricted *L-R* flat fuzzy numbers. However, there is no method in the literature to find the product of unrestricted *L-R* flat fuzzy numbers.

In this paper, firstly the product of unrestricted *L-R* flat fuzzy numbers is proposed and then with the help of proposed product, a new method (named as Mehar's method) is proposed for solving fully FLP problems. It is also shown that the fully FLP problems which can be solved by the existing methods can also be solved by the Mehar's method. However, such fully FLP problems in which some or all the parameters are represented by unrestricted *L-R* flat fuzzy numbers can be solved by Mehar's method but can not be solved by any of the existing methods.

1. Introduction

Linear programming is one of the most frequently applied operation research techniques. Although, it has been investigated and expanded for more than six decades by many researchers and from the various point of views, it is still useful to develop new approaches in order to better fit the real world problems within the framework of linear programming.

In conventional approach, parameters of linear programming models must be well defined and precise. However, in real world environment, this is not a realistic assumption. Usually, the value of many parameters of a linear programming model is estimated by experts. Clearly, it can not be assumed the knowledge of experts is so precise. Since, Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environments, a number of researchers have exhibited their interest to solve the FLP problems [2–8] and fully FLP problems [9–15].

On the basis of deep study of the existing methods for solving fully FLP problems, it can be concluded that there is no method in the literature for solving fully FLP problems in which some or all the parameters are represented by unrestricted *L-R* flat fuzzy numbers.

This paper is organised as follows: In Section 2, some basic definitions and arithmetic operations of *L-R* flat fuzzy numbers are presented. In Section 3, limitations of the existing method [9] are pointed out. In Section 4, product of unrestricted *L-R* flat fuzzy numbers is introduced. In Section 5, a new method, named as Mehar's method, is proposed to find the fuzzy optimal solution of fully FLP problems. In Section 6, advantages of the Mehar's method over the existing methods are discussed and to illustrate the Mehar's method numerical example is solved. Obtained results are discussed in Section 7. Conclusions are discussed in Section 8.

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2. Preliminaries

In this section, some basic definitions and arithmetic operations of L-R flat fuzzy numbers are presented.

2.1. Basic definitions

In this section, some basic definitions of L-R flat fuzzy numbers are presented.

Definition 2.1 [16]. A function $L : [0,\infty) \rightarrow [0,1]$ (or $R : [0,\infty) \rightarrow [0,1]$) is said to be reference function of fuzzy number if and only if

(i) $L(0) = 1$ (or $R(0) = 1$)
(ii) L (or R) is non-increasing on $[0,\infty)$.

Definition 2.2 [16]. A fuzzy number $\widetilde{A}$, defined on universal set of real numbers $\mathbb{R}$, denoted as $(m,n,\alpha,\beta)_{LR}$, is said to be an LR flat fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} L(\frac{m-x}{\alpha}) & x \leqslant m, \alpha > 0, \\ R(\frac{x-n}{\beta}) & x \geqslant n, \beta > 0, \\ 1 & m \leqslant x \leqslant n. \end{cases}$$

Definition 2.3 [14]. An L-R flat fuzzy number $\widetilde{A} = (m,n,\alpha,\beta)_{LR}$ is said to be non-negative L-R flat fuzzy number if $m - \alpha \geqslant 0$ and is said to be non-positive L-R flat fuzzy number if $n + \beta \leqslant 0$.

Definition 2.4 [14]. An L-R flat fuzzy number $\widetilde{A} = (m,n,\alpha,\beta)_{LR}$ is said to be unrestricted L-R flat fuzzy number if $m - \alpha$ is a real number.

Definition 2.5 [16]. Let $\widetilde{A} = (m,n,\alpha,\beta)_{LR}$ be an L-R flat fuzzy number and λ be a real number in the interval $[0,1]$ then the crisp set, $A_\lambda = \{x \in X : \mu_{\widetilde{A}}(x) \geqslant \lambda\} = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$, is said to be λ-cut of $\widetilde{A}$.

Definition 2.6 [16]. Let $\widetilde{A}_1 = (m_1,n_1,\alpha_1,\beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2,n_2,\alpha_2,\beta_2)_{LR}$, be any L-R flat fuzzy numbers then $\widetilde{A}_1 = \widetilde{A}_2$ iff $m_1 = m_2, n_1 = n_2, \alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$.

2.2. Arithmetic operations

In this section, the arithmetic operations between L-R flat fuzzy numbers are presented [16].

Let $\widetilde{A}_1 = (m_1,n_1,\alpha_1,\beta_1)_{LR}$, $\widetilde{A}_2 = (m_2,n_2,\alpha_2,\beta_2)_{LR}$ be any L-R flat fuzzy numbers and $\widetilde{A}_3 = (m_3,n_3,\alpha_3,\beta_3)_{RL}$ be any R-L flat fuzzy number. Then,

(i) $\widetilde{A}_1 \oplus \widetilde{A}_2 = (m_1+m_2, n_1+n_2, \alpha_1+\alpha_2, \beta_1+\beta_2)_{LR}$
(ii) $\widetilde{A}_1 \ominus \widetilde{A}_3 = (m_1-n_3, n_1-m_3, \alpha_1+\beta_3, \beta_1+\alpha_3)_{LR}$
(iii) If $\widetilde{A}_1$ and $\widetilde{A}_2$ both are non-negative, then
$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m_1m_2, n_1n_2, m_1\alpha_2+\alpha_1m_2-\alpha_1\alpha_2, n_1\beta_2+\beta_1n_2+\beta_1\beta_2)_{LR}$
(iv) If $\widetilde{A}_1$ is non-positive and $\widetilde{A}_2$ is non-negative, then
$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m_1n_2, n_1m_2, \alpha_1n_2-m_1\beta_2+\alpha_1\beta_2, \beta_1m_2-n_1\alpha_2-\beta_1\alpha_2)_{LR}$
(v) If $\widetilde{A}_1$ is non-negative and $\widetilde{A}_2$ is non-positive, then
$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (n_1m_2, m_1n_2, n_1\alpha_2-\beta_1m_2+\beta_1\alpha_2, m_1\beta_2-\alpha_1n_2-\alpha_1\beta_2)_{LR}$
(vi) If $\widetilde{A}_1$ and $\widetilde{A}_2$ both are non-positive, then
$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (n_1n_2, m_1m_2, -n_1\beta_2-\beta_1n_2-\beta_1\beta_2, -m_1\alpha_2-\alpha_1m_2+\alpha_1\alpha_2)_{LR}$
(vii) $\lambda\widetilde{A}_1 = \begin{cases} (\lambda m_1, \lambda n_1, \lambda\alpha_1, \lambda\beta_1)_{LR} & \lambda \geqslant 0 \\ (\lambda n_1, \lambda m_1, -\lambda\beta_1, -\lambda\alpha_1)_{RL} & \lambda < 0 \end{cases}$.

There also exist another formula [16] for the product of such L-R flat fuzzy numbers in which the spreads are smaller as compared to the mean values:

(i) If $\widetilde{A}_1$ and $\widetilde{A}_2$ both are non-negative, then
$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m_1m_2, n_1n_2, m_1\alpha_2+\alpha_1m_2, n_1\beta_2+\beta_1n_2)_{LR}$