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Interactive fuzzy random two-level linear programming based on level sets and fractile criterion optimization

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ABSTRACT

In this paper, assuming cooperative behavior of the decision makers, two-level linear programming problems under fuzzy random environments are considered by introducing α-level sets of fuzzy random variables and defining an α-stochastic two-level linear programming problem for guaranteeing the degree of realization of the problem. Through a new decision making model proposed based on the fractile criterion optimization model considering fuzzy goals, the α-stochastic two-level programming problem can be equivalently transformed into a deterministic problem. Interactive fuzzy programming to derive a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method.

1. Introduction

In the real world, we often encounter situations where there are two decision makers in an organization with a hierarchical structure, and they make decisions in turn or at the same time so as to optimize their objective functions. Such decision making situations can be formulated as a two-level programming problem [46]; one of the decision makers first makes a decision, and then the other who knows the decision of the opponent makes a decision.

In the context of two-level programming, the decision maker at the upper level first specifies a strategy, and then the decision maker at the lower level specifies a strategy so as to optimize the objective with full knowledge of the action of the decision maker at the upper level. In conventional multi-level mathematical programming models employing the solution concept of Stackelberg equilibrium, it is assumed that there is no communication among decision makers, or they do not make any binding agreement even if there exists such communication [4,28,54,55]. Compared with this, for decision making problems in such as decentralized large firms with divisional independence, it is quite natural to suppose that there exists communication and some cooperative relationship among the decision makers [46,47].

In order to deal with such cooperative two-level programming problems, Lai [17] and Shih et al. [53] proposed solution concepts for two-level linear programming problems or multi-level ones such that decisions of decision makers in all levels are sequential and all of the decision makers essentially cooperate with each other. In their methods, the decision makers identify membership functions of the fuzzy goals for their objective functions, and in particular, the decision maker at the upper level also specifies those of the fuzzy goals for the decision variables. The decision maker at the lower level solves a fuzzy programming problem with a constraint with respect to a satisfactory degree of the decision maker at the upper level. Unfortunately, there is a possibility that their method leads a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and those of the decision variables.

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To overcome such a problem in their methods, by eliminating the fuzzy goals for the decision variables, Sakawa et al. have proposed interactive fuzzy programming for two-level or multi-level linear programming problems to obtain a satisfactory solution for decision makers [49,50]. The subsequent works on two-level or multi-level programming have been appearing [1,18,29,32,39,44–47,51,52,56].

However, to utilize two-level programming for resolution of conflict in decision making problems in real-world decentralized organizations, it is important to realize that simultaneous considerations of both fuzziness [34–38,60,61,64,66,67] and randomness [5,11,42,43,57,58] would be required. Fuzzy random variables, first introduced by Kwakernaak [16], have been developing [15,24,30], and an overview of the developments of fuzzy random variables was found in [9]. Studies on linear programming problems with fuzzy random variable coefficients, called fuzzy random linear programming problems, were initiated by Wang and Qiao [59], Qiao, Zhang and Wang [31] as seeking the probability distribution of the optimal solution and optimal value. Optimization models for fuzzy random linear programming problems were first considered by Luhandjula et al. [25,27] and further developed by Liu [22,23] and Rommelfanger [33]. A brief survey of major fuzzy stochastic programming models was found in the paper by Luhandjula [26]. As we look at recent developments in the fields of fuzzy random programming, we can see continuing advances [2,3,8,10,13,14,19–21,33,40,41,48,62,63,65].

Under these circumstances, in this paper, assuming cooperative behavior of the decision makers, we consider solution methods for two-level linear programming problems involving fuzzy random variables. By introducing α-level sets of fuzzy random variables, an α-stochastic two-level linear programming problem for guaranteeing the degree of realization of the problem is defined. Considering vague natures of the decision makers' judgment, it is assumed that decision makers may have fuzzy goals for each of the objective functions in the α-stochastic two-level linear programming problem. Having elicited the membership functions which well represent the fuzzy goals of the decision makers at both levels, we propose a new decision making problem through the fractile criterion optimization model [12] together with the equivalent transformation of the α-stochastic two-level linear programming problem into the problem to maximize the satisfaction degree for each fuzzy goal. Following the proposed model, the transformed stochastic two-level programming problem can be reduced to a deterministic one. Interactive fuzzy programming to obtain a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented. It is shown that all of the problems to be solved in the proposed interactive fuzzy programming can be easily solved by the simplex method or the combined use of the bisection method and the simplex method. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method.

2. Fuzzy random two-level linear programming problems

Fuzzy random variables, first introduced by Kwakernaak [16], have been defined in various ways [15,16,24,30]. For example, as a special case of fuzzy random variables given by Kwakernaak, Kruse and Meyer [15] defined a fuzzy random variable as follows.

Definition 1 (*Fuzzy random variable*). Let (Ω, B, P) be a probability space, $F(\mathcal{R})$ the set of fuzzy numbers with compact supports and X a measurable mapping $\Omega \to F(\mathcal{R})$. Then X is a fuzzy random variable if and only if given $\omega \in \Omega$, $X_\alpha(\omega)$ is a random interval for any $\alpha \in (0, 1]$, where $X_\alpha(\omega)$ is an α-level set of the fuzzy set $X(\omega)$.

Although there exist some minor differences in several definitions of fuzzy random variables, fuzzy random variables are considered to be random variables whose observed values are fuzzy sets.

In this paper, we deal with two-level linear programming problems involving fuzzy random variable coefficients in objective functions formulated as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & z_1(\boldsymbol{x}_1,\boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{11}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{12}\boldsymbol{x}_2 \\ \underset{\text{for DM2}}{\text{minimize}} & z_2(\boldsymbol{x}_1,\boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{21}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{22}\boldsymbol{x}_2 \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leqslant \boldsymbol{b} \\ & \boldsymbol{x}_1 \geqslant \boldsymbol{0}, \quad \boldsymbol{x}_2 \geqslant \boldsymbol{0} \end{array}\right\} \tag{1}$$

It should be emphasized here that randomness and fuzziness of the coefficients are denoted by the "dash above" and "wave above" i.e., "$^-$" and "$^\sim$", respectively. In this formulation, $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the DM at the upper level (DM1), $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the DM at the lower level (DM2), $z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the objective function for DM1 and $z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the objective function for DM2. In (1), $\tilde{\bar{\boldsymbol{C}}}_{lj},\ l = 1,2,\ j = 1,2$ are vectors whose elements $\tilde{\bar{C}}_{ljk},\ k = 1,2,\ldots,n_j$ are fuzzy random variables characterized by the following membership function:

$$\mu_{\tilde{\bar{C}}_{ljk}}(\tau) = \begin{cases} L\left(\frac{\bar{d}_{ljk}-\tau}{\bar{\beta}_{ljk}}\right), & \text{if } \tau \leqslant \bar{d}_{ljk} \\ R\left(\frac{\tau-\bar{d}_{ljk}}{\bar{\gamma}_{ljk}}\right), & \text{otherwise} \end{cases}$$