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Research Article

Controller design approach based on linear programming

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ABSTRACT

This study explains and demonstrates the design method for a control system with a load disturbance observer. Observer gains are determined by linear programming (LP) in terms of the Routh–Hurwitz stability criterion and the final-value theorem. In addition, the control model has a feedback structure, and feedback gains are determined to be the linear quadratic regulator. The simulation results confirmed that compared with the conventional method, the output estimated by our proposed method converges to a reference input faster when a load disturbance is added to a control system. In addition, we also confirmed the effectiveness of the proposed method by performing an experiment with a DC motor.

1. Introduction

Control theory is mainly classified as classical control theory and modern control theory. Classical control theory is designed on the basis of a transfer function of the controlled plant, and it has an output feedback structure. Usually, the parameters of the controller are chosen based on the experience of the control engineer. The proportional-integral-derivative (PID) controller, which is the most representative method, is now widely employed in industrial applications. The controller is designed so that various indices such as the steady-state error, damping coefficient, gain margin, and phase margin are satisfied.

On the other hand, modern control theory is described by dynamic variables known as state variables, which represents a state-space model of the controlled plant in the time-domain. Control performance is commonly quantified as a cost function, and an optimal control system is designed on the basis of this function. In modern control theory, it is possible to treat not only the input and output variables, but also the state variables representing the state of the controlled plant. Thus, we can perform more rigorous analysis, enabling the design of more complex controlled plants. Position, velocity, and acceleration are examples of state variables. Position can be measured by a detector known as a potentiometer, which is relatively inexpensive. On the other hand, sensors for velocity and acceleration are generally more expensive. In our study, we adopt an estimator known as an "observer" [1–4]. The basic idea for an observer was proposed by Luenberger in 1964 [5]. An observer estimates the velocity and acceleration from a plant's input and output, i.e., the position information. Regardless of whether the initial value of the state variables contains an error or whether the plant system is unstable, the estimation error of the state variables converges to zero within a finite time.

The advanced technologies for an observer have been proposed by several researchers. For instance, based on the state variable filter, an adaptive observer for a linear system was proposed by Kreisselmeier [6] and Luders and Narendra [7]. Shimada and Phaoharuhansa proposed an adaptive observer for a stable plant [8]. Besancon et al., Paesa et al., Mousavi-Aghdam et al., and Zhao et al. proposed the extension of an adaptive observer for a nonlinear system [9–12]. Furthermore, Zhu proposed a method using a low-dimensionalized adaptive observer [13]. When the control system has state-feedback structures, these techniques have reduced the influence of the state estimation error and disturbance by estimating an unknown parameter or disturbance with an adaptive observer.

Zhang et al. proposed the equivalent model of an observer in which a plant's output-side disturbance is canceled [1,2]. A conventional disturbance observer is composed of an expanded system with state variables and disturbance. The disturbance observer can estimate the disturbance and decrease it by using a feedback control action to the plant input. On the other hand, Zhang et al. theoretically derived an observer that removes the disturbance applied to the output side. Also, their method

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adjusted a linear quadratic regulator (LQR) [14–16] by providing a specific region so that the optimal solutions found by the Riccati equation are all stable. The effectiveness of their method is confirmed in both simulations and experiments. The robustness appears to be excellent because their design method is based on a linear quadratic regulator. However, the residual disturbance at steady-state remains very small.

In this paper, we describe a controller design technique that allows for quick convergence to the reference input even if the disturbance becomes mixed with the system. The key is to obtain the observer gains based on linear programming (LP) [17–19]. In particular, we obtain constraints from the Routh–Hurwitz stability criterion [20–23] and a cost function by the final-value theorem [23]. On the basis of these two terms, a method for determining the observer gains can be attributed to linear programming. Although we need to solve fractional programming with an absolute value, this can be calculated by linear programming.

This paper is organized as follows: in Section 2, we describe how to obtain optimal state-feedback gains with an LQR. In Section 3, the idea of observer error separation is described, after which the constraints and the cost function for the observer gains with LP are formulated. In Section 4, the effectiveness of the method is confirmed by simulation results and experimental results, which show that the proposed method converges to the reference input faster than the method proposed by Zhang et al. [1,2]. In Section 5, we briefly look at an application to higher-order plants. In Section 6, we present a conclusion.

2. Review of PID controller design with an LQR [2]

The proposed method requires a state-feedback structure, and it is assumed that the plant model is a single-input single-output (SISO) structure. Thus, the feedback from motor states, i.e., speed and drive current, actually acts as a proportional controller and derivative controller, respectively. If we select the cascaded controller to be the integrator, the total control law is equivalent to a PID controller.

We use the optimal control theory to select PID parameters [24–26] such that the cost function with the weighted sum of the state vector and control energy is minimized [14–16]. Then, the PID control system is shown in Fig. 1.

Let the state-space model of the plant $G(s)$ be

$$\dot{\boldsymbol{x}}_1(t)=\boldsymbol{A}_1\boldsymbol{x}_1(t)+\boldsymbol{b}_1u_1(t), \tag{1a}$$

$$y_1(t)=\boldsymbol{c}_1\boldsymbol{x}_1(t), \tag{1b}$$

where $\boldsymbol{x}_1(t)\in\Re^{n\times1}$ is the state vector, $\dot{\boldsymbol{x}}_1(t)\in\Re^{n\times1}$ is a differential of the state vector, $u_1(t)\in\Re^{1\times1}$ is an input, $y_1(t)\in\Re^{1\times1}$ is a plant model output without a disturbance, and $\boldsymbol{A}_1\in\Re^{n\times n}$, $\boldsymbol{b}_1\in\Re^{n\times1}$, and $\boldsymbol{c}_1\in\Re^{1\times n}$ are the constant matrices of appropriate dimensions for a plant. Let the entire system output $y(t)\in\Re^{1\times1}$ be the sum of the motor output and load disturbance, i.e.,

$$y(t)=y_1(t)+d(t), \tag{2}$$

where $d(t)\in\Re^{1\times1}$ is an output-side disturbance. On the other hand, the state-space model of the controller $G_c(s)$ can be written as

$$\dot{\boldsymbol{x}}_2(t)=\boldsymbol{A}_2\boldsymbol{x}_2(t)+\boldsymbol{b}_2u_2(t), \tag{3a}$$

$$y_2(t)=\boldsymbol{k}_2\boldsymbol{x}_2(t), \tag{3b}$$

$$u_2(t)=r(t)-y(t), \tag{3c}$$

where $\boldsymbol{x}_2(t)\in\Re^{n\times1}$, $u_2(t)\in\Re^{1\times1}$, $y_2(t)\in\Re^{1\times1}$, $r(t)\in\Re^{1\times1}$, $\boldsymbol{A}_2\in\Re^{n\times n}$, $\boldsymbol{b}_2\in\Re^{n\times1}$, and $\boldsymbol{k}_2\in\Re^{1\times n}$ are constant matrices of appropriate dimensions for the controller. The basic idea is to convert a PID tuning problem to that of an optimal control design. To achieve this aim, we formulate the closed-loop cascaded systems (1) and (3) with $d(t)=0$ into an augmented system such as

$$\dot{\boldsymbol{x}}_e(t)=\boldsymbol{A}_e\boldsymbol{x}_e(t)+\boldsymbol{b}_eu_1(t)+\boldsymbol{E}_er(t), \tag{4a}$$

$$y_e(t)=y_1(t)=\boldsymbol{c}_e\boldsymbol{x}_e(t), \tag{4b}$$

where

$$\boldsymbol{A}_e=\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\\ -\boldsymbol{b}_2\boldsymbol{c}_1 & \boldsymbol{A}_2\end{bmatrix},\quad \boldsymbol{b}_e=\begin{bmatrix}\boldsymbol{b}_1\\ \boldsymbol{0}\end{bmatrix},\quad \boldsymbol{E}_e=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{b}_2\end{bmatrix},$$

$$\boldsymbol{x}_e=\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\end{bmatrix},\quad \boldsymbol{c}_e=[\boldsymbol{c}_1\ \boldsymbol{0}],$$

and $(\boldsymbol{A}_1,\boldsymbol{b}_1,\boldsymbol{c}_1)$ and $(\boldsymbol{A}_2,\boldsymbol{b}_2)$ are the state-space models of the plant and the controller, respectively.

The resulting state-feedback LQR for the augmented system (4) can be chosen as

$$u_1(t)=-\boldsymbol{K}_e\boldsymbol{x}_e(t)=-\boldsymbol{k}_1\boldsymbol{x}_1(t)-\boldsymbol{k}_2\boldsymbol{x}_2(t). \tag{5}$$

For the SISO second order system, $\boldsymbol{K}_e=[\boldsymbol{k}_1,\boldsymbol{k}_2]=[k_{11},k_{12},k_2]\in\Re^{1\times3}$, k_{11} is the gain of the proportional controller, k_{12} is the gain of the derivative controller, and k_2 is the gain of the integral controller.

To determine the optimal feedback gains, the vector $\boldsymbol{K}_e$ is calculated using the optimal control method, which is given by

$$J=\int_0^\infty\{\boldsymbol{x}_e(t)^\top\boldsymbol{Q}\boldsymbol{x}_e(t)+Ru_1(t)^2\}\,dt, \tag{6}$$

where $\boldsymbol{Q}$ is a diagonal matrix with positive elements ($\boldsymbol{Q}\geq0$) and R is a positive coefficient ($R>0$). Q and R represent the relative importance of the states variation and control energy consumption, respectively. The optimal state-feedback gain $\boldsymbol{K}_e$, which minimizes the performance index J in (6), is given by

$$\boldsymbol{K}_e=R^{-1}\boldsymbol{b}_e^\top\boldsymbol{P}. \tag{7}$$

The symmetric matrix $\boldsymbol{P}>0$ is the solution of the Riccati equation [15,16]. The designed closed-loop system becomes

$$\begin{bmatrix}\dot{\boldsymbol{x}}_1(t)\\ \dot{\boldsymbol{x}}_2(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}_1-\boldsymbol{b}_1\boldsymbol{k}_1 & -\boldsymbol{b}_1k_2\\ -\boldsymbol{b}_2\boldsymbol{c}_1 & \boldsymbol{A}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\end{bmatrix}+\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{b}_2\end{bmatrix}r(t), \tag{8a}$$

$$y_1(t)=\boldsymbol{c}_1\boldsymbol{x}_1(t). \tag{8b}$$

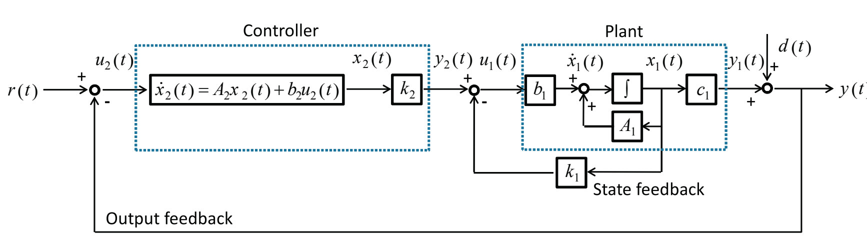

Fig. 1. Block diagram of a PID control system.