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Fuzzy linear programming approach to multiattribute decision making with multiple types of attribute values and incomplete weight information

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ABSTRACT

In the classical Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP), the decision maker (DM) gives the pair-wise comparisons of alternatives with crisp truth degree 0 or 1. However, in the real world, DM is not sure enough in all comparisons and can express his/her opinion with some fuzzy truth degree. Thus, DM's preferences are given through pair-wise comparisons of alternatives with fuzzy truth degrees, which may be represented as trapezoidal fuzzy numbers (TrFNs). Considered such fuzzy truth degrees, the aim of this paper is to develop a new fuzzy linear programming technique for solving multiattribute decision making (MADM) problems with multiple types of attribute values and incomplete weight information. In this method, TrFNs, real numbers, and intervals are used to represent the multiple types of decision information. The fuzzy consistency and inconsistency indices are defined as TrFNs due to the alternatives' comparisons with fuzzy truth degrees. Hereby a new fuzzy linear programming model is constructed and solved by the possibility linear programming method with TrFNs developed in this paper. The fuzzy ideal solution (IS) and the attribute weights are then obtained. The distances of alternatives from the fuzzy IS can be calculated to determine their ranking order. The implementation process of the method proposed in this paper is illustrated with a strategy partner selection example. The comparison analyzes show that the method proposed in this paper generalizes the classical LINMAP, fuzzy LINMAP and possibility LINMAP.

1. Introduction

The classical Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP) developed by Srinivasan and Shocker [1] is an effective and a simple method for solving multiattribute decision making (MADM) problems. The LINMAP is based on pair-wise comparisons of alternatives given by the decision maker (DM) and generates the best compromise alternative as the solution that has the shortest distance to the ideal solution (IS).

In the LINMAP, all the decision data are known precisely or given as crisp values. However, under many conditions, crisp data are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information are vague or fuzzy in nature and as such it may not be appropriate to represent them by accurate numerical values. A more realistic approach could be to use fuzzy sets [2], intuitionistic fuzzy (IF) sets (IFSs) [3,4] and linguistic variables [5–7] to model human judgments. Therefore, extending the LINMAP to suit the fuzzy or IF environments is of a great importance for scientific researches and real applications [8–12]. Li and Yang [8] used linguistic variables to assess an alternative on qualitative attributes. These linguistic variables are transformed into positive triangular fuzzy numbers (TFNs) and hereby the LINMAP was developed for multiattribute group decision making (MAGDM) with linguistic variables. Xia et al. [9] also applied linguistic variables to capture fuzziness in decision information and processes by means of a fuzzy decision matrix. The linguistic variables were represented by trapezoidal fuzzy numbers (TrFNs). A fuzzy LINMAP was proposed for solving MADM problems under fuzzy environments. Li and Sun [10] transformed linguistic variables into TFNs and extended the LINMAP

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for solving MAGDM problems with linguistic variables and incomplete weight preference information. Using IFSs to express the attribute values, Li [11] and Li et al. [12] respectively extended the LINMAP for MADM and MAGDM under IF environment.

The real-life decision making problems often involve multiple different attribute values. In the assessment process, alternatives have to be evaluated on different attributes, which may be qualitative and quantitative. Due to the DM's knowledge area and the nature of evaluated attributes, the assessments provided by the DM may be measured in different formats such as real numbers, intervals, TrFNs, linguistic variables and IFSs. Consequently, MADM problems with multiple types of attribute values have drawn much attention from a wide spectrum of disciplines [13–16]. With ever increasing complexity in many real situations, there are often some challenges for the DM to provide precise and complete preference information due to time pressure, lack of knowledge (or data) and the DM's limited expertise about the problem domain. In other words, usually weights are totally unknown or partially known a priori. Namely, weight preference information in MADM problems is usually incomplete. Recently, there are some methods for solving MADM and MAGDM problems with incomplete preference information [10,17–21]. However, the methods [10,17–21] are not applicable to the MADM problems with multiple types of attribute values and the methods [13–16] cannot deal with the MADM problems with incomplete weight information.

In the classical LINMAP [1] and the fuzzy LINMAP [8–12], the DM gives the pair-wise comparisons of alternatives in the form of the ordered pairs with crisp truth degree 0 or 1. However, in the real world, DM is not sure enough in all comparisons and can express his/her opinion with a fuzzy degree of truth. Sadi-Nezhad and Akhtari [22] considered the fuzzy truth degree as a TFN and transformed the information of decision matrix into TFNs. They proposed the possibility LINMAP in group decision making. Wan and Li [23] firstly introduced IFSs to depict the fuzzy truth degrees of alternatives' comparison and developed a fuzzy LINMAP for solving heterogeneous MADM problems. As Wan and Li [23] pointed out that there exist some big mistakes in the definitions, notations, operations, and possibilistic programming model in [22].

To overcome the above disadvantages, the aim of this paper is to extend the possibilistic LINMAP for solving MADM problems with multiple types of attribute values and incomplete weight information. The main works lie in two aspects. On the one hand, a TrFN permits two parameters to represent the most possible values while a TFN uses a single parameter to represent the most possible value. Namely, a TFN is a special case of a TrFN. Therefore, a TrFN is not only valuable for modeling imprecision but also easy to reflect the ambiguous nature of subjective judgments. TrFNs are used to capture fuzzy truth degree information about pair-wise comparisons of alternatives in this study. On the other hand, TrFNs, intervals and real numbers are used to represent the multiple types of attribute values. Considered the comparisons of alternatives with fuzzy truth degrees, the fuzzy consistency and inconsistency indices are defined as TrFNs. The fuzzy IS and attribute weights are then obtained through constructing a new fuzzy linear programming model which is solved by the possibility linear programming method with TrFNs developed in this paper.

Compared with the existing LINMAP [1,8–12,22,23], the method proposed in this paper has the following differences and advantages:

(1) The classical LINMAP [1] and the fuzzy LINMAP [8–12] did not consider the DM's preferences on the pair-wise comparisons of alternatives with fuzzy degrees of truth. In other words, they only considered the crisp truth degrees 0 or 1. In fact, due to the complexity of decision problems and fuzziness of human's thinking, there exist some uncertainty and fuzziness when DM gives the pair-wise comparisons of alternatives. Consequently, it is very natural and reasonable to introduce fuzzy numbers to represent the information of fuzzy truth degrees. This is a great innovation and main motivation of our paper.
(2) This paper utilizes TrFNs to represent the fuzzy truth degrees which can better reflect the ambiguous nature of subjective judgments on the pair-wise comparisons of alternatives given by DM, while [22] used TFNs to express fuzzy truth degrees. Furthermore, the MADM problems studied in this paper involve multiple types of attribute values, whereas that studied in [22] considered only single type of attribute values.
(3) To obtain the fuzzy IS and vector of attribute weights, the linear programming model constructed in this paper is fuzzy. We technically develop a new method to solve this kind of fuzzy linear programming models with TrFNs. However, though the constructed linear programming model in [22] is fuzzy linear programming with TFNs, Sadi-Nezhad and Akhtari did not propose any new method to solve it.
(4) Under some conditions, the proposed method in this paper can be reduced to the classical LINMAP [1], fuzzy LINMAP [8–12] and the possibility LINMAP [22]. Namely, the LINMAP [1,8–12,22] is a special case of this paper's method (see Sections 5.2 and 5.3 in detail).
(5) Wan and Li [23] considered the hesitance degree of alternatives' comparison and represented the fuzzy truth degrees as IFSs, while this paper expresses the fuzzy truth degrees as TrFNs. The former constructed fuzzy linear programming with IFSs, whereas the latter constructed fuzzy linear programming with TrFNs. Both fuzzy linear programming models have completely different solving methods. Therefore, the decision principles and motivations for both papers are remarkably different.

The rest of the paper is structured as follows. In Section 2, the distance between TrFNs is defined and the interval objective programming models are introduced. In Section 3, the fuzzy MADM problems with multiple types of attribute values are described, and the normalization method is discussed as well as incomplete weight information structures. In Section 4, in order to solve such MADM problems, a new fuzzy linear programming model is constructed and solved by the developed new possibility linear programming method with TrFNs. The proposed method is illustrated with a real strategy partner selection example and comparison analyzes are conducted in Section 5. Conclusion is given in Section 6.

2. Distances for trapezoidal fuzzy numbers and interval objective programming

In this section, some preliminaries about trapezoidal fuzzy numbers and interval objective programming are firstly introduced.