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Limit analysis of masonry walls by rigid block modelling with cracking units and cohesive joints using linear programming

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ABSTRACT

In this paper a rigid block model with cracking units and cohesive joints is developed for limit analysis of masonry structures using linear programming. The model is applicable to in-plane loaded unreinforced masonry block walls with regular textures and provides as output failure loads and collapse mechanisms. A simplified micro-modelling approach is adopted, based on the discretization of masonry units into triangular blocks separated by contact interfaces. Tensionless contact interfaces are used to model dry joints and cohesive contacts are introduced for mortar joints and internal unit interfaces. Failure modes at contact interfaces involve cracking, crushing and sliding. An iterative solution procedure is used to model non-associative flow rule in sliding and to take into account no-tension behaviour of cohesive joints in case of cracking. The modelling approach is validated and applied to different structural examples from the literature, including unconfined and confined masonry panels and a masonry beam. To show the accuracy of the proposed model and the improvement of the predicted response, the obtained results are compared with experimental tests and with the outcomes of standard rigid block models using a single block per masonry unit and tensionless interfaces.

1. Introduction

In rigid block limit analysis, masonry walls are discretized into a number of rigid bodies interacting at contact interfaces. The discretization of masonry structures into rigid blocks represents a well consolidated approach in the literature, which was already used in the static theories of the past centuries and then considered by Kooharian and Heyman in their studies on the application of the limit analysis theorems to historical construction systems [1,2].

The formulation of the limit analysis problem by mathematical programming is generally expressed at the block and contact level in terms of equilibrium equations between external and internal forces as well as by kinematic equations for the geometric compatibility of contact strain rates and block degrees of freedom [3–5]. The behaviour at interfaces is characterized by yield functions of contact static variables and by flow rules, relating contact strain rates and plastic multipliers. The term 'plastic' refers to the rigid perfectly-plastic model which is assumed for the behaviour of contact interfaces, involving arbitrarily large strains at constant yield stress levels. Under this assumption and considering associative flow rules, as specified in the following, it can be shown that the theorems of plastic limit analysis originally developed for steel structures can be applied to masonry structures as well, although the forms of inelasticity that masonry actually exhibits are quasi-brittle [1].

The limit analysis problem may result into a linear (LP) or mixed complementarity program (MCP), depending on the form of yield functions and corresponding flow rules, namely associative and non-associative, with unique or multiple solutions. In case of multiple solutions, several procedures have been investigated to find the minimum load factor associated to the corresponding limit analysis problem [6–12].

In computational limit analysis of masonry panels using micro-modelling, each masonry unit is usually assumed to be rigid and is modelled with a single block, expanded by half of the mortar thickness in case of mortar joints [13]. The behaviour of dry joints, mortar joints and mortar-unit interfaces is lumped into a zero-thickness contact interface (Fig. 1). Failure modes are defined at interfaces which represent the potential fracture lines. It is also worth noting that the tensile strength and cohesion at interfaces are usually assumed equal to zero for both dry and mortared masonry panels, mainly due to the fact that this modelling approach has been usually adopted for ancient structures with poor mortar quality [9,13].

The application of rigid block limit analysis to the above mentioned micro-models (herein referred to as 'standard') usually provides a good agreement between experimental and numerical outcomes, but there are some cases in which such an approach

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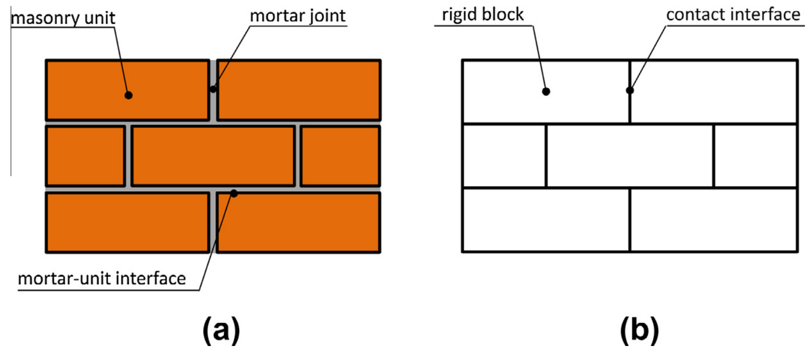

Fig. 1. (a) Representation of a masonry wall with mortar joints; (b) standard rigid-block micro-model.

may lead to incorrect prediction of the collapse load for in-plane loaded panels. It is the case of collapse mechanisms involving the cracking of the masonry units, that the standard model does not take into account, and/or of masonry walls with strong mortar joints, whose cohesive behaviour significantly contribute to the structural response.

The problem of modelling cracking units in rigid block limit analysis was faced by Orduña and Lourenço at first [14]. Indeed the authors, following the results of experimental tests on a shear wall panel made of dry jointed stone blocks, introduced a cracking unit in the numerical model using two rigid blocks to model the masonry stone which was observed to fail. That significantly improved the prediction of the response in terms of both the collapse load and the failure mechanism, resulting in a more accurate and effective numerical outcomes.

Considering the valuable improvement of predicted response obtained in [14] with the introduction of a cracking unit and having in mind the possibility to detect automatically the potential masonry units subject to cracking, a micro-modelling approach for the application of rigid block limit analysis to masonry panels has been developed and herein presented. The modelling approach can be used for in-plane loaded unreinforced masonry block structures with regular textures and provides as output failure load and collapse mechanism.

The main aim of the present study is to evaluate the effectiveness of the implemented approach in predicting the collapse load and failure modes of different structural examples from the literature, concerning in-plane loaded masonry panels with dry and mortar joints. The study comes within a research activity focused on the use of iterative and sequential linear programming for limit analysis of masonry structures [12,15,16].

Different simplified assumptions have been made to model the potential failure of the masonry units and the cohesive behaviour of mortar joints, mainly concerning the discretization approach and the modelling of failure mechanisms.

In the proposed formulation each masonry unit is modelled as an assemblage of six triangular rigid blocks separated by internal contact interfaces, which represent the potential crack paths of the masonry unit itself. Tensionless contact interfaces are used to model dry joints and cohesive contacts are introduced for mortar joints and internal unit interfaces.

Similar to the standard rigid block modelling approach, failure modes are defined at interfaces and include cracking, crushing and sliding.

Non-associative flow rule in sliding failure is considered, according to the iterative procedure of linear programs proposed in [12].

A key aspect of the proposed modelling approach is the implementation of a simple algorithm, based on iterative linear programming, which takes into account the no-tension behaviour for cohesive contact interfaces subject to cracking.

The basic idea was to include in the model the tensile strength of uncracked cohesive contacts and at the same time to reduce to zero the contribution of the plastic energy dissipated by cohesive forces at interfaces subject to opening, which could produce different collapse mechanisms and an overestimation of the predicted failure load with respect to experimental tests, as shown in this study. It should be noted that the use of iterative procedures to take into account no-tension behaviour of material is well consolidated in the literature. Castigliano was among the first to implement an iterative procedure in his study on the analysis of the Mosca bridge in Turin [17]. Recently, a similar procedure has been used by Galassi et al. [18] for non-linear analysis of masonry structures using rigid body spring models.

Of course, the developed iteration algorithm represents a considerable simplification of the brittle behaviour of materials in tension failure and refined modelling approaches can be found in the literature to take into account softening behaviour by mathematical programming [19].

On the other side, it should be pointed out that the combined use of: (i) micro-modelling, which allows to take into account geometric parameters such as size, arrangement and orientation of masonry units that may strongly influence local and global response of wall panels [20–22]; (ii) non-associative behaviour in sliding failure; (iii) the above-mentioned iterative procedure for the auto-detection of cracking units is a specific feature of the proposed formulation compared with the existing ones.

On the basis of the proposed formulation, a computer based program has been developed which provides as output the failure loads and a plot of the corresponding collapse mechanism.

The paper is organized as follows. The implemented modelling approach, the limit analysis formulation and the adopted solution procedure are presented in Sections 2-4.

In Section 5 a validation study has been carried out on the unconfined shear wall panels investigated by Orduña and Lourenço [14]. This application is also used to show the results obtained at the different steps of the implemented procedure.

Section 6 presents the application of the developed formulation to other literature case studies which involve different failure modes of the units and of the panels. The selected case studies are a deep masonry beam, a full scale perforated masonry panel and two confined shear walls with and without opening. These last case studies involve special issues in limit analysis relevant to the modelling of displacement restrains at the top of the wall panels. The results of the numerical simulations on the different wall samples are presented and compared with experimental tests and with the outcomes of the standard rigid block model.