Optimal trajectory planning for trains under fixed and moving signaling systems using mixed integer linear programming

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The optimal trajectory planning problem for multiple trains under fixed block signaling systems and moving block signaling systems is considered. Two approaches are proposed to solve this optimal control problem for multiple trains: the greedy approach and the simultaneous approach. In each solution approach, the trajectory planning problem is transformed into a mixed integer linear programming (MILP) problem. In particular, the objective function considered is the energy consumption of trains and the nonlinear train model is approximated by a piece-wise affine model. The varying line resistance, variable speed restrictions, and maximum traction force, etc. are also included in the problem definition. In addition, the constraints caused by the leading train in a fixed or moving block signaling system are first discretized and then transformed into linear constraints using piecewise affine approximations resulting in an MILP problem. Simulation results comparing the greedy MILP approach with the simultaneous MILP approach show that the simultaneous MILP approach yields a better control performance but requires a higher computation time. Moreover, the performance of the proposed greedy and the proposed simultaneous MILP approach is also compared with that of the greedy and the simultaneous pseudospectral method, where the pseudospectral method is a state-of-the-art method for solving optimal control problems. The results show that the energy consumption and the end time violations of the greedy MILP approach are slightly larger than those of the greedy pseudospectral method, but the computation time is one to two orders of magnitude smaller. The same trend holds for the simultaneous MILP approach and the simultaneous pseudospectral method.

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1. Introduction

Nowadays, the energy efficiency of transportation systems is becoming more and more important because of the rising energy prices and environmental concerns. Rail traffic plays a significant role for the sustainability for transportation systems, since it can provide safe, fast, punctual, and comfortable services (Peng, 2008). The reduction of energy consumption is one of the key objectives of railway systems because energy consumption is one of the major expenses in operational cost, which is about 13–16% of the annual operation and maintenance cost of railway systems in China (Ding, Bai, Liu, & Mao, 2009). Therefore, even a small improvement in energy saving is attractive to the railway operators since it can save a large amount of money. Some driver assistance systems have been developed to assist drivers to drive the train optimally, such as FreightMiser (Howlett & Pudney, 1995), Metromiser (Howlett & Pudney, 1995), and driving style manager (Franke, Meyer, & Terwiesch, 2002). With the development of modern railway systems, an automatic train operation system plays a key role in ensuring accurate stopping, operation punctuality, energy saving, and riding comfort (Peng, 2008). The railway control center or automatic train operation systems are responsible for solving the trajectory planning problems based on the information collected by train monitoring systems, such as line resistance, speed limits, maximum traction and braking forces.

In the literature, the research on the optimal control of train operations began in the 1960s and is aimed at solving the trajectory planning problem for a train running from one station to another. Since it has significant effects for energy saving, punctuality, etc., various approaches were proposed for the trajectory planning problem. These approaches can be grouped into two main categories: analytical solutions and numerical optimization. For analytical solutions, the maximum principle is applied and it results in four optimal regimes (i.e., maximum traction, cruising, coasting, and maximum braking) (Howlett,
2. Train model and the MILP approach

In this section, the formulation of the optimal control problem and the MILP approach we proposed in Wang et al. (2013) are summarized.

2.1. Optimal control problem

A continuous-space mass-point model is often used in the literature on train optimal control (Franke, Terwiesch, & Meyer, 2003), which can be described as follows (Liu & Golovicher, 2003):

\[
m\frac{d^2 \dot{s}}{ds^2} = u(s) - R_0(v) - R(s, v),
\]

\[
\frac{df}{ds} = \frac{1}{2}E,
\]

where \( m \) is the mass of the train, \( \rho \) is a factor to consider the rotating mass (Hansen & Pachl, 2008), \( E \) is the kinetic energy per mass unit, which is equal to \( 0.5v^2 \), \( v \) is the velocity of the train, \( s \) is the position of the train, \( u \) is the control variable, i.e. the traction or braking force, which is bounded by the maximum traction force \( u_{\text{max}} \) and the maximum braking force \( u_{\text{min}} \). So \( u_{\text{min}} \leq u(s) \leq u_{\text{max}} \). \( R_0(v) \) is the basic resistance including roll resistance and air resistance, and \( R(s, v) \) is the line resistance caused by track grade, curves, and tunnels. See Wang et al. (2013) for more details.

The kinetic energy per mass unit \( \dot{E} = 0.5v^2 \) and time \( t \) are chosen as the states and the position \( s \) is taken as the independent variable for the train model as in Franke et al. (2003). The trajectory planning problem for trains can then be formulated as (Wang et al., 2011)

\[
J = \int_{s_{\text{start}}}^{s_{\text{end}}} \max(0, u(s)) \, ds
\]

s.t.

\[
\begin{align*}
\min \leq u(s) \leq \max, \\
0 < \dot{E}(s) \leq \dot{E}_{\text{max}}(s), \\
\dot{E}(s)_{\text{start}} = \dot{E}_{\text{end}}, \\
\dot{t}(s)_{\text{start}} = 0, \quad \dot{t}(s)_{\text{end}} = T,
\end{align*}
\]

and the train model (1), where the objective function \( J \) is the energy consumption without regenerative braking; \( \dot{E}_{\text{max}}(s) \) is equal to \( 0.5V_{\text{max}}^2(s) \) where \( V_{\text{max}}(s) \) is the maximum allowable velocity, which depends on the train characteristics and line conditions, and as such it is usually a piecewise constant function of the coordinate \( s \) (Khmelnitsky, 2000; Liu & Golovicher, 2003); \( s_{\text{start}}, \dot{E}_{\text{start}} \) and \( t_{\text{start}} \) are the position, the kinetic energy per mass, and the departure time at the beginning of the route; \( s_{\text{end}}, \dot{E}_{\text{end}}, \) and \( t_{\text{end}} \) are the position, the kinetic energy per mass, and the arrival time at the end of the route, respectively, where the scheduled running time \( T \) is given by the timetable or the

The remainder of this paper is structured as follows. In Section 2, the train model and the MILP approach for a single train are summarized based on Wang et al. (2013). Section 3 introduces the principle of railway signaling systems, i.e. the FBS system and the MBS system. Section 4 formulates the constraints for the following train caused by the leading train under an FBS system and shows how to include these constraints into the MILP formulation. The constraints caused by the MBS system are considered and included in the MILP problem in Section 5. Section 6 illustrates the calculation of the optimal trajectories using the data from Beijing Yizhuang subway line. We conclude with a short discussion of some topics for future work in Section 7.
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