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A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers

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ABSTRACT

Linear programming (LP) is a widely used optimization method for solving real-life problems because of its efficiency. Although precise data are fundamentally indispensable in conventional LP problems, the observed values of the data in real-life problems are often imprecise. Fuzzy sets theory has been extensively used to represent imprecise data in LP by formalizing the inaccuracies inherent in human decision-making. The fuzzy LP (FLP) models in the literature generally either incorporate the imprecisions related to the coefficients of the objective function, the values of the right-hand-side, and/or the elements of the coefficient matrix. We propose a new method for solving FLP problems in which the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers. We convert the FLP problem into an equivalent crisp LP problem and solve the crisp problem with the standard primal simplex method. We show that the method proposed in this study is simpler and computationally more efficient than two competing FLP methods commonly used in the literature.

1. Introduction

Linear programming (LP) is a mathematical technique for optimal allocation of scarce resources to several competing activities on the basis of given criteria of optimality. Precise data are fundamentally indispensable in conventional LP problems. However, the observed values of the data in real-life problems are often imprecise. Fuzzy sets theory has been used to handle imprecise data in LP by generalizing the notion of membership in a set. Essentially, each element in a fuzzy set is associated with a point-value selected from the unit interval [0,1]. The fundamental challenge in fuzzy LP (FLP) is to construct an optimization model that can produce the optimal solution with imprecise data.

The theory of fuzzy mathematical programming was first proposed by Tanaka et al. Tanaka et al. [1] based on the fuzzy decision framework of Bellman and Zadeh [2]. Zimmerman [3] introduced the first formulation of FLP to address the impreciseness of the parameters in LP problems with fuzzy constraints and objective functions. Zimmerman [3] constructed a crisp

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model of the problem and obtained its crisp results using an existing algorithm. He then used Bellman and Zadeh [2] interpretation that a fuzzy decision is a union of goals and constraints and fuzzified the problem by considering subjective constants of admissible deviations for the goal and the constraints. Finally, he defined an equivalent crisp problem using an auxiliary variable that represented the maximization of the minimization of the deviations on the constraints. FLP is by far the most widely used method by practitioners for constrained optimization problems with fuzzy data [4–10].

We propose a simplified new method for solving FLP problems in which the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers. We show that the optimal solution of the FLP problem can be found simply by solving an equivalent crisp LP problem.

The remainder of this paper is organized as follows. We review the relevant FLP literature in Section 2. In Section 3, we review some necessary concepts and backgrounds on fuzzy arithmetic. We then formulate the FLP problem proposed by Ganesan and Veeramani [11] in Section 4. In Section 5, we present our proposed FLP method. We present our conclusions and future research directions in Section 6.

2. Literature review

There are generally five FLP classifications in the literature:

- Zimmermann [12] has classified FLP problems into two categories: (1) symmetrical problems and (2) non-symmetrical problems. In a symmetrical problem there is no difference between the weight of the objectives and constraints while in the non-symmetrical problems, the objectives and constraints are not equally important and have different weights [13].
- Leung [14] has classified FLP problems into four categories: problems with (1) precise objective and fuzzy constraints; (2) fuzzy objective and precise constraints; (3) fuzzy objective and fuzzy constraints; and (4) robust programming.
- Luhandjula [15] has classified FLP models into three categories: (1) flexible programming; (2) mathematical programming with fuzzy parameters; and (3) fuzzy stochastic programming.
- Inuiguchi et al. [16] have classified FLP models into six categories: (1) flexible programming; (2) possibilistic programming; (3) possibilistic LP using fuzzy max; (4) robust programming; (5) possibilistic programming with fuzzy preference relations; and (6) possibilistic LP with fuzzy goals.
- Kumar et al. [17] have classified FLP problems into two categories: FLP problems with (1) inequality constraints and (2) equality constraints. Some authors [18–20] have proposed different methods for solving FLP problems with inequality constraints where the FLP problem is first converted into a crisp LP problem and then the resulting crisp LP problem is solved to find the fuzzy optimal solution for the original FLP problem. Other authors [21,22] have proposed methods for solving FLP problems with equality constraints which are generally approximate.

In the past four decades, numerous researchers have studied various properties of FLP problems and proposed different models for solving LP problems with fuzzy data. Tanaka et al. [1] first proposed the theory of fuzzy mathematical programming and Zimmerman [3] first formulated and solved the FLP problem. Tanaka and Asai [23] proposed a possibilistic LP formulation where the coefficients of the decision variables were crisp while the decision variables were fuzzy numbers. Verdegay [24] presented the concept of a fuzzy objective based on the fuzzification principle and used this concept to solve FLP problems. Herrera et al. [25] examined the fuzzified version of the mathematical problem assuming that the coefficients are given by fuzzy numbers and the relations in the definition of the feasible set are also fuzzy.

Zhang et al. [26] proposed a FLP with fuzzy numbers for the objective function coefficients. They showed how to convert FLP problems into multi-objective optimization problems with four objective functions. Stanciulescu et al. [27] proposed a FLP model with fuzzy coefficients for the objective function coefficients and the constraints. Their model uses fuzzy decision variables with a joint membership function instead of crisp decision variables and linked the decision variables together to sum them to a constant.

Ganesan and Veeramani [11] proposed a FLP model with symmetric trapezoidal fuzzy numbers. They proved fuzzy analogues for some important LP theorems and derived a solution for the FLP problems without converting them into crisp LP problems. Ebrahimnejad [28] showed that the method proposed by Ganesan and Veeramani [11] stops in a finite number of iterations and proposed a revised version of their method that was more efficient and robust in practice. He also proved the absence of degeneracy and showed that if an FLP problem has a fuzzy feasible solution, it also has a fuzzy basic feasible solution and if an FLP problem has an optimal fuzzy solution, it also has an optimal fuzzy basic solution.

Mahdavi-Amiri and Nasseri [29] proposed a FLP model where a linear ranking function was used to order trapezoidal fuzzy numbers. They established the dual problem of the LP problem with trapezoidal fuzzy variables and deduced some duality results to solve the FLP problem directly with the primal simplex tableau. Mahdavi-Amiri and Nasseri [30] developed some methods for solving the FLP problems by introducing certain auxiliary problems. They applied a linear ranking function to order trapezoidal fuzzy numbers and deduced some duality results by establishing the dual problem of the LP problem with trapezoidal fuzzy variables. Ebrahimnejad et al. [31] introduced a new primal–dual algorithm for solving FLP problems by using the duality results proposed by Mahdavi-Amiri and Nasseri [30]. Ebrahimnejad [32] also generalized the concept of