Information Sciences 271 (2014) 159–178

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Optimal solution of multi-objective linear programming with inf-$\rightarrow$ fuzzy relation equations constraint

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ARTICLE INFO

Article history:
Received 15 August 2012
Received in revised form 8 July 2013
Accepted 15 February 2014
Available online 27 February 2014

Keywords:
Fuzzy linear programming
Optimal solution
Inf-$\rightarrow$ composition
Fuzzy relation equations constraint

ABSTRACT

This paper aims to solve the problem of multiple-objective linear optimization model subject to a system of inf-$\rightarrow$ composition fuzzy relation equations, where $\rightarrow$ is *R*-, *S*- or *QL*-implications generated by continuous Archimedean *t*-norm (*s*-norm). Since the feasible domain of inf-$\rightarrow$ relation equations constraint is nonconvex, these traditional mathematical programming techniques may have difficulty in computing efficient solutions for this problem. Therefore, we firstly investigate the solution sets of a system of inf-$\rightarrow$ composition fuzzy relation equations in order to characterize the feasible domain of this problem. And then employing the smallest solution of constraint equation, we yield the optimal values of linear objective functions subject to a system of inf-$\rightarrow$ composition fuzzy relation equations. Secondly, the two-phase approach is applied to generate an efficient solution for the problem of multiple-objective linear optimization model subject to a system of inf-$\rightarrow$ composition fuzzy relation equations. Finally, a procedure is represented to compute the optimal solution of multiple-objective linear programming with inf-$\rightarrow$ composition fuzzy relation equations constraint. In addition, three numerical examples are provided to illustrate the proposed procedure.

1. Introduction

It is well known that fuzzy relation equations are associated with knowledge and inference from a body knowledge [42]. Since Sanchez [48] proposed the resolution of fuzzy relation equation, different fuzzy relation equations have been extensively studied by many researchers. The problem to determine an unknown fuzzy relation R on universe of discourses $U \times V$ such that $R \circ A = B$, where A and B are given fuzzy set on U and V, respectively, and $\circ$ is an composite operation of fuzzy relations, is called the problem of fuzzy relation equations. How to compute the solutions of fuzzy relation equations is one of the most appealing subjects in fuzzy set theory. Fuzzy relation equations can be roughly classified into two categories according to composite operations, that is, sup-t composition fuzzy relation equations and inf-$\rightarrow$ composition fuzzy relation equations. Recently, many researchers investigated the solvability of the sup-t fuzzy relation equations, and then various methods have been developed to detect the minimal solutions for the sup-t fuzzy relation equations [2,19,26,32,44,46,50–52,55,56,59,63,65]. It is worth to mention that Li and Fang provided a good overview of fuzzy relation equations [29]. Peeva proposed algorithms for inverse problem resolution of fuzzy linear systems of equations in some

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http://dx.doi.org/10.1016/j.ins.2014.02.110

BL-algebras [43]. Molai represented an algorithm find the solution set of sup-product fuzzy relation equation using the $L \circ U$-factorization [39].

Fuzzy relation equations have much wider application fields, such as fuzzy control, discrete dynamic systems, knowledge engineering, identification of fuzzy systems, prediction of fuzzy systems, decision-making, fuzzy information retrieval, fuzzy pattern recognition, image compression and reconstruction, and so on. Among the problems related to fuzzy relation equations, optimization of objective functions subject to fuzzy relation equations regions are interesting. It is well known that the linear programming problem is concerned with optimizing linear objective function of real variables on a feasible domain with linear equality or inequality constraints. In practice, we need to translate the linear programming problem to a fuzzy setting, for instance by replacing the linear constraints by a set of fuzzy relation equations. Many research efforts have been devoted to minimizing a linear objective function subject to a consistent system of sup-*t*-norm relation equations [6,10,11,15–17,27,28,34,45,49,57,58]. For example, Guu and Wu provided a procedure for solving linear optimization problems with the max-Archimedean *t*-norm fuzzy relation equation constraint [15]. Li and Fang showed that the linear optimization problem subject to a system of sup-*t* equations can be reduced to a 0–1 integer optimization problem in polynomial time [27]. Wu et al. translated the problem into an equivalent integer programming problem which is then solved by means of the branch and bound method [58,61]. Ghodousian and Khorram presented an algorithm to generate optimal solutions of linear objective function optimization with respect to the fuzzy relation inequalities defined by max–min composition [13]. Some research efforts have also been devoted to wider generalizations of the problem. Wu et al. represented an efficient method to optimize a linear fractional programming problem [60]. Dempe and Ruziyeva considered the fuzzy linear optimization problem with fuzzy coefficients [7]. Fan et al. developed a generalized fuzzy linear programming method to reflect ambiguous information in actual management problems [8]. Dubey et al. studied linear programming problems involving interval uncertainty modeled using intuitionistic fuzzy set [9]. Ghodousian and Khorram investigated a linear optimization problem with fuzzy relation inequality constraints [12].

Compared to the multi-objective linear programming model, the fuzzy multi-objective linear programming model can effectively reflect the uncertain information and decision makers' subjective preference or interactive choice in practical applications. Employing the fuzzy numbers, Maeda converted the fuzzy linear programming problem into two objective linear programming problems [37]. Zhang et al. extended Maeda's result by formulating the fuzzy linear programming problem as a multi-objective programming problem with four objectives [66]. Pal et al. solved fuzzy multi-objective linear programming problem by the use of a goal programming approach [41]. Arikan and Gungor considered multi-objective problems with all fuzzy parameters [1]. Jiménez and Bilbao investigated multi-objective linear programming problems where the aspiration levels fixed for each objective are imprecise [20]. Nehi and Hajmohamadi solved the fuzzy linear programming problem by the ranking function method [40]. Rommelfanger represented a general interactive solution process for solving multicriteria linear programming systems with fuzzy linear systems [47]. Kaur and Kumar showed a new method to find the fuzzy optimal solution of multi-objective fuzzy linear programming formulation of fully fuzzy minimal cost flow problem [21]. Toksarı used a Taylor series to solve fuzzy multi-objective linear fractional programming problem [53]. Chakraborty and Sandipan Gupta presented a solution procedure for multi-objective linear fractional programming problem [5]. Xu and Zhou established linear fuzzy multi-objective models with expected objectives and chance constraints, and then applied the proposed models to an earth-rock work allocation problem [64]. Khalili-Damghani et al. used Technique for Order Preference by Similarity to Ideal Solution to reduce the multi-objective problem into a bi-objective problem [22].

Wang firstly explored the fuzzy multi-objective linear programming subject to max-*t*-norm composition fuzzy relation equations [54]. Guu et al. proposed a two-phase method to solve fuzzy multiple linear programming problem under max-*t*-norm fuzzy relation equations constraint [18]. Loetamonphong et al. provided a genetic algorithm to find the Pareto optimal solutions for a nonlinear multi-objective optimization problem with fuzzy relation equations constraint [33]. Khorram and Zarei considered a multiple objective optimization model subject to a system of fuzzy relation equations with max-average composition [23]. Lu and Fang presented a genetic algorithm to resolve an optimization model with a nonlinear objective function subject to a system of fuzzy relation equations [35]. Gong et al. presented a novel evolutionary algorithm that interacts with a decision maker during the optimization process to obtain the most preferred solution [14].

Fuzzy reasoning, as a generalization of classical logical inference, is better than binary logic reasoning in realistic environment of human knowledge. The results of fuzzy inference completely depend on the choice of fuzzy sets of fuzzy antecedent, fuzzy consequences and fuzzy connectives linking fuzzy antecedents and fuzzy consequences. The representation of fuzzy if-then rules is non-unique, which results in fuzzy relation equations with various composite operations in the realization of fuzzy if-then rules. Given the importance of Boolean-type implications (mainly contain *R*-, *S*- and *QL*-implications) in fuzzy if-then rules, it is no trivial to study the inf–$\rightarrow$ composition fuzzy relation equations. Inf-$\rightarrow$ composition fuzzy relation equations were firstly studied by Miyakoshi and Shimbo [38], where the implication is arbitrary a *R*-implication. Similar to sup-*t* composition fuzzy relation equations, some ways have been developed to search the maximal solutions in order to determine the complete solution sets of inf-$\rightarrow$ composition fuzzy relation equations [30,31,36,62].

In this paper, we investigate the following multi-objective linear programming problem with *k* objective functions:

$$\text{Min} \quad \mathbf{z}(\mathbf{x}) = C\mathbf{x} \tag{1}$$

$$\text{s.t.} \quad A \circ_{\rightarrow} \mathbf{x} = \mathbf{b} \tag{2}$$