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Fuzzy decision making for fuzzy random multiobjective linear programming problems with variance covariance matrices

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A B S T R A C T

We focus on fuzzy random multiobjective linear programming problems with variance covariance matrices, and propose a fuzzy decision making method to obtain a satisfactory solution. In the proposed method, it is assumed that the decision maker has fuzzy goals for both permissible levels and the corresponding objective functions for a probability maximization model or a fractile optimization model, and that such fuzzy goals are quantified by eliciting the corresponding membership functions. The satisfactory solution based on the fuzzy decision is obtained using the bisection method and the convex programming technique.

1. Introduction

In the real world, decisions often have to be made under uncertainty. In computer science, stochastic and fuzzy programming approaches have been developed to deal with decision problems involving uncertainty. In stochastic programming approaches [2,4,5,12], two stage problems and chance constrained programming models have been investigated in various ways, and have been extended to multiobjective stochastic programming problems (MOSPs) [24,25]. A variety of fuzzy programming problems have been formulated and investigated [18,21,34], and as a natural extension, a multiobjective fuzzy programming technique was first proposed by Zimmermann [33]. Many other methods have since been proposed [22,34].

In addition, mathematical programming problems with fuzzy random variables [17] have also been proposed [13,19,27]. The concepts of such mathematical programming problems include both probabilistic uncertainty and fuzzy problems simultaneously. Because such fuzzy random programming problems are commonly ill-defined, utilizing both stochastic and fuzzy techniques is necessary to construct an effective decision making model.

To deal with probabilistic uncertainty and fuzzy problems simultaneously, hybrid approaches that combine stochastic and fuzzy programming techniques have been proposed [10]. Sakawa et al. [23] proposed an interactive method for multiobjective fuzzy linear programming problem with random variable coefficients, and Katagiri et al. [14,16] proposed an interactive method for multiobjective linear programming problems with fuzzy random variable coefficients. However, decision makers must specify permissible levels for a probability maximization model or a fractile optimization model. Predefining such values can be difficult because conflicts exist among permissible levels and the corresponding objective function values. To circumvent the problem of decision makers specifying such permissible levels in advance, Yano and Sakawa [31,32]

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proposed an interactive method for the specialized formulation of multiobjective linear programming problems with fuzzy random variable coefficients to obtain a satisfactory solution, where it is assumed that any coefficient in the objective function is expressed by "only one" fuzzy random variable. The proposed method can be easily solved, because the formulated problems can be reduced to linear programming problems. However, the assumption that any coefficient in the objective function can be expressed by only one fuzzy random variable may not be realistic. Recently, Xu et al. [30] formulated two types of fuzzy random multiobjective portfolio selection models, in which it is assumed that future returns of risky securities are fuzzy random variables with variance covariance matrices. They proposed a method to obtain the efficient frontier and subsequently applied their method to the Chinese stock market. Ammar [1] also formulated fuzzy random multiobjective quadratic programming problems with fuzzy coefficients and fuzzy decision variables. He also applied his method to the portfolio optimization problem. In addition, Xu and Liu [29] developed a fuzzy random multiobjective model for inventory problems, in which all inventory costs, purchasing, and selling prices in the objective functions and constraints are assumed to be fuzzy random variables. They proposed algorithms to determine optimal quantities for facilitating maximum profit and minimal wastage costs simultaneously. Katagiri and Sakawa [15] proposed an interactive decision making method for multiobjective fuzzy random programming problems using a level set concept and a probability maximization model. Biswas and De [3] formulated multiobjective chance constrained programming problems involving log-normally distributed fuzzy random variables and they proposed a fuzzy goal programming approach to obtain a satisfactory solution. More recently, Li and Xu [28] formulated multiobjective portfolio selection model with fuzzy random returns and developed the corresponding genetic algorithm to obtain a compromised portfolio strategy.

In this study, we propose a fuzzy decision making method for fuzzy random multiobjective linear programming problems (FRMOLPs) with variance covariance matrices. In the proposed method, the decision maker needs to specify the membership functions of the fuzzy goals for the original objective functions and permissible levels of a possibility measure for a probability maximization model or specify the membership functions of the fuzzy goals for the original objective functions and permissible probability levels for a fractile optimization model, but is not required to specify the permissible levels themselves. A satisfactory solution can be easily obtained using the fuzzy decision [22,34] to integrate these membership functions. The remainder of this paper is organized as follows. In Section 2, we discuss using a possibility measure [7] to formulate FRMOLPs with variance covariance matrices as MOSPs [14,16]. In Section 3, we explain the transformation of MOSPs to probability maximization and fractile optimization model-based programming problems [14,16]. In addition, we define two types of Pareto optimal solutions and investigate the relationships that exist between them. In Section 4, a fuzzy decision making method for FRMOLPs with variance covariance matrices is proposed through a probability maximization model. We also present an algorithm based on the bisection method and the convex programming technique that we use to obtain a satisfactory solution. In Section 5, a fuzzy decision making method for FRMOLPs with variance covariance matrices is proposed through a fractile optimization model. In addition, we show that the maxmin problem through a fractile optimization model is equivalent to a maxmin problem through a probability maximization model. In Section 6, we formulate a crop planning problem as a fuzzy random multiobjective programming problem with variance covariance matrices, and apply the proposed fuzzy decision making method to this problem. To show the efficiency of the proposed method, we compare the optimal solution of the proposed method with the corresponding solutions obtained from the probability maximization and fractile optimization models proposed by Katagiri et al. [14,16]. Finally, we present conclusions in Section 7.

2. Fuzzy random multiobjective programming problems with variance covariance matrices

In this section, we focus on a multiobjective programming problem involving fuzzy random variable coefficients in objective functions.

[FRMOLP]

$$\min \tilde{\bar{C}}\boldsymbol{x} = (\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}, \ldots, \tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}) \tag{1}$$

subject to

$$\boldsymbol{x} \in X \overset{\text{def}}{=} \{A\boldsymbol{x} \leqslant \boldsymbol{b}, \boldsymbol{x} \geqslant \boldsymbol{0}\} \tag{2}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T$ is an n dimensional decision variable column vector, A is an $(m \times n)$ coefficient matrix, and $\boldsymbol{b} = (b_1, \ldots, b_m)^T$ is an m dimensional column vector.

$$\tilde{\bar{\boldsymbol{c}}}_i = (\tilde{\bar{c}}_{i1}, \ldots, \tilde{\bar{c}}_{in}), \quad i = 1, \ldots, k, \tag{3}$$

is a set of coefficient vectors of objective function $\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}$, whose elements are fuzzy random variables. The concept of fuzzy random variable $\tilde{\bar{c}}_{ij}$ is precisely defined in [17,20,23]), and the symbols "–" and "$\sim$" denote randomness and fuzziness respectively.

To deal with the objective functions $\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x},\ i = 1, \ldots, k$, Katagiri et al. [14,16] proposed an LR-type fuzzy random variable that can be regarded as a special type of a fuzzy random variable. Under the occurrence of each elementary event $\omega, \tilde{\bar{c}}_{ij}(\omega)$ is a realization of an LR-type fuzzy random variable $\tilde{\bar{c}}_{ij}$, which is an LR fuzzy number [7] with a membership function that is defined as follows.