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A linearized formulation of AC multi-year transmission expansion planning: A mixed-integer linear programming approach

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ABSTRACT

This paper presents a method in expansion planning of transmission systems using the AC optimal power flow (AC-OPF). The AC-OPF provides a more accurate picture of power flow in the network compared to the DC optimal power flow (DC-OPF) that is usually considered in the literature for transmission expansion planning (TEP). While the AC-OPF-based TEP is a mixed-integer *nonlinear* programming problem, this paper transforms it into a mixed-integer *linear* programming environment. This transformation guarantees achievement of a global optimal solution by the existing algorithms and software. The proposed model has been successfully applied to a simple 3-bus power system, Garver's 6-bus test system, 24-bus IEEE reliability test system (RTS) as well as a realistic power system. Detailed case studies are presented and thoroughly analyzed. Simulations show the effectiveness of the proposed method on the TEP.

1. Introduction

Transmission expansion planning (TEP) addresses the problem of augmenting transmission lines of an existing transmission network; the objective is to optimally serve a growing electric load while satisfying a set of economical, technical and reliability constraints [1]. In general, the TEP is considered as making a stochastic decision on when (the time), where (the location), and which types of transmission lines to be installed. In [2,3], a classification scheme categorizes the subjects published in this area.

A mixed-integer linear programming (MILP) is used in [4], for the TEP problem that considers power losses. We suggest a linearization method for the AC-TEP based on the method in [4]. However, both active and reactive powers are included in our proposed formulation. In [5], a multi-year TEP model is presented using a discrete evolutionary particle swarm optimization approach. Aguado et al. [6], present a novel TEP model that considers a multi-year planning horizon in a competitive electricity market. Both the TEP and generation expansion planning (GEP) problems are analyzed together in [7,8]. Also, transmission switching (TS) is investigated in [9] for the TEP showing that the TS could improve the capacity expansion planning model as well as reducing the total planning cost. A meta-heuristic and holistic approach is presented in [10] for the TEP which has been tested on a realistic power system. In [11], a scenario-based multi-objective model is presented for multi-stage TEP where the non-dominated sorting genetic algorithm (NSGA-II) is used to overcome the difficulties in solving the non-convex mixed-integer optimization problem. A multi-objective framework is presented in [12] for the TEP in deregulated environment. Recently, intermittent energy resources have been experiencing a rapid growth in power generation around the world. Therefore, new challenges are introduced to integrate the renewable energy sources (RES) to the power grid. There are many published papers which focus on this newly main issue [13–15].

The planner of power system should deal with many uncertainties during the planning process such as load uncertainty, uncertainty in prices, market rules and etc. A multistage TEP problem including available transfer capability (ATC) is modeled in [16] that takes load uncertainty into account by considering several scenarios generated by Monte Carlo simulation. In [17], the TEP is studied by considering the load uncertainty using benders decomposition. Since this paper is focused on transforming a mixed-integer nonlinear programming (MINLP) problem into a MILP, uncertainties have not been considered in this paper. However, the presented model can easily be extended for taking uncertainties into account.

The surveyed literatures use the DC-OPF for solving the TEP problem which is not completely suitable due to ignoring reactive power. However, there have been some published papers which use the AC-OPF to solve the TEP problem [18–21]. This paper proposes an approach for transmission planning based on the AC-OPF,

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Nomenclature

Indices

g	index of generators
i, j	indices of buses
l	index of lines
m	index of blocks used for piecewise linearization
t	index of sub-periods (times)

Sets

B	set of all buses
CL	set of all candidate lines
CL_i	set of all candidate lines connected to bus i
EL	set of all existing lines
EL_i	set of all existing lines connected to bus i
G	set of all generators
SP	set of all sub-periods

Constants

$AP_{L.l}^{\max}$	maximum apparent power flow of line l
I	interest rate
IC_l	investment cost of candidate line l
M	number of blocks used for piecewise linearization
nc	number of candidate lines
ng	number of generators
$P_{D,it}$	active power demand at bus i in time t
$Q_{D,it}$	reactive power demand at bus i in time t
$P_{G,g}^{\min}$	minimum active power of generator g
$P_{G,g}^{\max}$	maximum active power of generator g
$Q_{G,g}^{\min}$	minimum reactive power of generator g
$Q_{G,g}^{\max}$	maximum reactive power of generator g
$\lvert V_i^{\min}\rvert$	minimum of the voltage magnitude at bus i
$\lvert V_i^{\max}\rvert$	maximum of the voltage magnitude at bus i
Y_{ij}^0, Y_{ij}	admittance of line ij for the existing and candidate lines, respectively. ($Y_{ij}^0 = G_{ij}^0 + jB_{ij}^0$, $Y_{ij} = G_{ij} + jB_{ij}$)
$\alpha_{ij,m}$	slope of the mth block of the voltage angle difference between corridor i and j
$\Delta\theta_{ij}$	maximum of each block width for corridor i and j
θ_{ref}	voltage phase angle for the slack bus ($\theta_{ref} = 0$)
σ	a constant to make investment and operation cost comparable
τ_{ij}, υ, ω_{ij}, ψ_1, ξ_{ij},	disjunctive parameters

Variables:

IC_t	investment cost in time t
OC_t	operation cost in time t
OF	objective function
$P_{G,gt}$	active power of generator g in time t
$P_{L_{i\to j},lt}^0$	active power flow of existing line l from bus i to bus j in time t
$P_{L_{i\to j}}, lt$	active power flow of candidate line l from bus i to bus j in time t
$Q_{G,gt}$	reactive power of generator g in time t
$Q_{L_{i\to j},lt}^0$	reactive power flow of existing line l from bus i to bus j in time t
$Q_{L_{i\to j}}, lt$	reactive power flow of candidate line l from bus i to bus j in time t
u_{lt}	binary variable related to the candidate lines in time t
$\lvert V_{it}\rvert$	voltage magnitude at bus i in time t
$\theta_{ij,mt}$	width of the mth angle block of corridor i and j in time t
θ_{ijt}^+, θ_{ijt}^-	positive variables used so as to eliminate the absolute function
θ_{it}	voltage angle at bus i in time t

providing a more accurate picture of both active and reactive power flows in the expanded power network in the future planning horizon. The novelty of this paper is the introduction of a MILP formulation using the AC-OPF approach so as to solve the expansion planning problem of transmission grid. In brief, an AC-OPF-based TEP is formulated, and linearized around the operating point in order to derive a MILP problem. Solving a MILP problem is a mature technology, where the MILP solvers can be embedded in many tools and applications. Moreover, some numerical examples are presented in which simulations are discussed accordingly. The whole linearization process provided in subsection “B” of section “II” that converts the non-linear AC approach to a MIP problem is novel and has not been previously presented. This paper contributes to the TEP by approximating the sine and cosine functions in power flow equations by their Taylor's series; then, the quadratic function is modeled using piecewise linear functions. Moreover, the inequality constraints for apparent powers of existing and candidate lines are transformed into a set of linear constraints. Numerical results confirm the contribution of the proposed method in comparison with the conventional solutions. Since the proposed optimization problem for solving the TEP is linear, the global optimal solution can be obtained easily by the available software. In addition, outcomes obtained by the proposed method are more accurate (due to taking reactive power into account) than those of the available conventional methods (due to ignoring reactive power by using the DC-OPF for solving the TEP). It should be emphasized that the ISO (independent system operator) is responsible for transmission expansion planning; the ISO aims at minimizing the investment cost plus the total payment to the generating companies.

2. Analysis and formulation of the TEP

Here the proposed method is formulated, presenting the TEP based on the AC-OPF using a MINLP that will be transformed into a MILP.

2.1. The AC-OPF-based TEP formulation

The objective function is the investment cost (IC) which is the construction cost of new lines and transformers (if any) plus the operation cost (OC). The OC includes the total cost of generation in the power system under study. Using an AC power flow, objective function of the TEP can be formulated as follows:

$$\text{Min } OF = \sum_{t\in SP}(1+I)^{-t}\left(\underbrace{\sum_{l=1}^{nc} u_{lt} IC_l}_{\text{Investment cost in time } t(IC_t)} + \sigma \underbrace{\sum_{g=1}^{ng} C_g P_{G,gt}}_{\text{Operation cost in time } t(OC_t)}\right) \quad (1)$$

The objective function (1) is subjected to the following equality and inequality constraints;

Equality constraints:

$$P_{G,it} - P_{D,it} = \sum\nolimits_{l\in EL_i} P_{L_{i\to j},lt}^0 + \sum\nolimits_{l\in CL_i} P_{L_{i\to j},lt} \quad \forall i,j \in B, \forall t \in SP \quad (2)$$