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MULTI-PARAMETER LINEAR PERIODIC SYSTEMS: SENSITIVITY ANALYSIS AND APPLICATIONS

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For stability analysis of linear periodic systems with more than one degree of freedom, the Floquet method is a general and valuable, practical method. In multi-parameter periodic systems, repeated numerical integration to obtain the Floquet matrix may be a limiting factor, and effective sensitivity analysis of stability characteristics is therefore needed. Analytical first and second order sensitivities of the Floquet matrix and its eigenvalues (multipliers) are presented in this paper. Some numerical applications are given. These include effective stabilization by proper change of parameters and optimal design with constraints on stability requirements.

1. INTRODUCTION

Stability analysis for even a single, second order, linear differential equation with periodic coefficients (Mathieu–Hill equation) is rather cumbersome, but different methods are available. Among these are the method of infinite determinants [1], the perturbation method [2], the Galerkin method [3], and the classic Floquet method (see reference [4] or [5]).

Few of these methods can, from a practical point of view, be extended to multi-degree-of-freedom (d.o.f.) systems, i.e., to coupled, second order, linear systems with matrices containing periodic coefficients. For such extensions see references [6–8]. It is concluded that the Floquet method is a general and practical method for systems with multi-d.o.f. See also references [9, 10].

Even with increasing computer power, the large number of numerical integrations required in this method limits the possibilities; so research with the goal of carrying out these integrations in the most effective way have recently been

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conducted, see references [11,12]. In the present paper, we shall include sensitivity analysis in the Floquet method and, by this means, get more information on each numerical integration performed. This sensitivity analysis is carried out analytically first by finding the derivatives of the Floquet matrix and then using these to study the behaviour of eigenvalues of this matrix in the complex plane. For early reference to sensitivity analysis for non-self-adjoint eigenvalue problems, see reference [13].

The contents of this paper are as follows. First, a short introduction to the Floquet method is given, followed by the necessary mathematics for deriving first and second order sensitivities, and finally – examples. The numerical examples show the versatility of sensitivity analysis with focus on effective stabilization by proper change of parameters and on optimal design of a beam with constraints on stability requirements.

2. THE FLOQUET METHOD AND THE FLOQUET MATRIX

In this section, we discuss the classical Floquet theory for stability of a system of linear, homogeneous, differential equations with periodic coefficients. Consider a system of linear, homogeneous, differential equations with periodic coefficients

$$\dot{\mathbf{x}} = \mathbf{G}(t)\mathbf{x}, \tag{2.1}$$

where $\mathbf{G}(t)$, $t \in \mathbb{R}$, is a real $(m \times m)$-matrix function. The vector $\mathbf{x}$ is a column vector of dimension m. Let $\mathbf{G}(t)$ be periodic with minimum period T. That is, T is the smallest positive number for which $\mathbf{G}(t + T) = \mathbf{G}(t)$ for all $t \in \mathbb{R}$.

Let $\mathbf{x}_1(t), \mathbf{x}_2(t), \ldots, \mathbf{x}_m(t)$ be any set of m solutions to the system (2.1), linearly independent for any $t \in \mathbb{R}$ (and thus for all $t \in \mathbb{R}$). The matrix $\mathbf{X}(t)$ with columns $\mathbf{x}_1(t), \mathbf{x}_2(t), \ldots, \mathbf{x}_m(t)$ is called a fundamental matrix. If $\mathbf{X}(0) = \mathbf{I}$ where $\mathbf{I}$ is the $(m \times m)$-identity matrix, $\mathbf{X}(t)$ is called a principal fundamental matrix. The fundamental matrix $\mathbf{X}(t)$ is non-singular for all $t \in \mathbb{R}$.

If $\mathbf{X}(t)$ is a principal fundamental matrix, the matrix given by

$$\mathbf{F} = \mathbf{X}(T) \tag{2.2}$$

is called the Floquet transition matrix or the monodromy matrix, see reference [14]. For brevity, we name it the Floquet matrix.

The Floquet matrix $\mathbf{F}$ can be computed in a single integration scheme, by numerically solving the system

$$\dot{\mathbf{y}} = \mathbf{H}\mathbf{y} \tag{2.3}$$

where $\mathbf{y}$ is a vector of dimension m^2 and $\mathbf{H}(t)$ is an $(m \times m)$-matrix of $(m \times m)$-submatrices

$$\mathbf{H} = \begin{bmatrix} \mathbf{G} & \mathbf{0} & \cdot & \mathbf{0} \\ \mathbf{0} & \mathbf{G} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \mathbf{0} \\ \mathbf{0} & \cdot & \mathbf{0} & \mathbf{G} \end{bmatrix}, \tag{2.4}$$