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Sensitivity analysis and optimal design of 3D frame structures for stress and frequency constraints

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Abstract

The present paper deals with the problem of determining the optimal joint positions and cross-sectional parameters of linearly elastic space frames with imposed stress and free frequency constraints. The frame is assumed to be acted on by different load systems, including temperature and self-weight loads. The stress state analysis includes tension, bending, shear, and torsion of beam elements. By a sequence of quadratic programming problems, the optimal design is attained. The sensitivity analysis of distinct as well as multiple frequencies is performed through analytic differentiation with respect to design parameters. Illustrative examples of optimal design of simple and medium complexity frames are presented, and the particular case of bimodal optimal solution is considered in detail. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

In this paper, we shall discuss the problem of sensitivity analysis and optimal design of frame structures for which both cross-sectional and configuration design parameters are to be determined from the solution. We shall impose the stress and free frequency constraints on the optimal design. The first constraint provides proper stress levels under specified loads, the other constraint is aimed to assume proper structure response under dynamic excitation for which the resonant conditions are avoided and proper frequency spectrum is obtained. The optimal design problems with free frequency constraints were treated in numerous papers (cf. [5,9,10] and references cited therein). Similarly, the sensitivity analysis for single and multiple eigenfrequencies was considered by numerous authors (cf. Wittrick [22], Masur and Mroz [11,12], Haug and Rousselet [4], Pedersen [16, 17], Mills-Curran [14], Mc Gee and Phan [13], Olhoff et al. [15], Krog and Olhoff [8]). The aim of this paper is to extend the previous analyses and consider variation of both cross-sectional and configurational parameters. It turns out that structures are much more sensitive to configuration changes, so search for optimal structure joint positions provides much more efficient designs. The use of stress and frequency constraints assures practical designs for which both static and dynamic responses are controlled.

One of the characteristic features of free frequency constrained designs is occurrence of multiple or nearly equal eigenvalues. Such coincidence of free frequencies is associated with structural symmetry or is induced by the evolution of eigenvalue spectrum due to redesign process toward an optimum with constraint set on the fundamental frequency. It is well known that multiple

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frequencies are not differentiable in the common sense (that is are not Frechet differentiable) and only directional sensitivity derivatives can be calculated. This fact creates some difficulties in finding sensitivities of multiple frequencies with respect to design parameters and in applying the effective gradient optimization techniques.

The present paper is devoted to development of an efficient method of sensitivity analysis and optimization of space frame structures with single and multiple eigenfrequencies. The sequential quadratic programming scheme is used with direct application of sensitivity derivatives. It was found that the applied method is reliable and accurate. The specific examples are concerned with a two-beam space frame, frame dome with 52 beam elements, and finally with a frame of mobile crane.

2. Sensitivity analysis for single and multiple eigenfrequencies

Consider a linear elastic discretized frame structure for which the static equilibrium equation takes the form

$$[S]\{D\} = \big\{\{A_1\} + \{A_2\} + \{A_3\}\big\} \quad (1)$$

where $[S]$ is the global stiffness matrix, $\{D\}$ is the vector of joint displacements, and $\{A_1\}$ is the vector of external joint loads, $\{A_2\}$ is the vector of distributed equivalent loads, $\{A_3\}$ is the vector of temperature equivalent loads. The state of free vibrations is governed by the symmetric eigenvalue problem

$$\big([S] - \lambda_j[M]\big)\big\{\Phi_j\big\} = 0, \quad j = 1, \ldots, n \quad (2)$$

where $[M]$ is the global mass matrix, $\lambda_j = \omega_j^2$ is the jth squared angular vibration frequency, $\{\Phi_j\}$ is the associated eigenmode and n denotes the number of degrees of freedom. The explicit forms of $[S]$ and $[M]$ are provided in the Appendix A. The eigenvalues are all real and can be ordered in the following manner:

$$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_j \leq \cdots \leq \lambda_n \quad (3)$$

The eigenvectors $\{\Phi_j\}$ are all $[S]$-orthogonal and $[M]$-normalized, so that

$$\{\Phi_i\}^{\mathrm{T}}[S]\big\{\Phi_j\big\} = \lambda_j\delta_{ij},$$

$$\{\Phi_i\}^{\mathrm{T}}[M]\big\{\Phi_j\big\} = \delta_{ij}, \quad i,j = 1, \ldots, n \quad (4)$$

The system stiffness and mass matrices are accumulated from beam element stiffness $[S_e]$ and mass matrices $[M_e]$, i.e., symbolically (omitting Boolean matrices) we have

$$[S] = \sum_e [C_{4e}]^{\mathrm{T}}[S_e][C_{4e}],$$

$$[M] = \sum_e [C_{4e}]^{\mathrm{T}}[M_e][C_{4e}] \quad (5)$$

The transformation matrix $[C_{4e}]$ is a block-diagonal matrix composed of four orthogonal transformation matrices $[C_e],[C_e]^{-1} = [C_e]^{\mathrm{T}}$ relating the beam element to the global Cartesian reference frame. Considering two basic types of constraints, namely frequency and stress constraints, further, selecting the set of design parameters contained in the design vector $\{X\}$, the optimization problem can be stated as follows. It is required to determine from a given range $[\{X\}_{\min},\{X\}_{\max}]$ the joint positions (shape parameters) as well as cross-sectional dimensions (size parameters) $\{X^*\}$ for which the frame mass attains its minimum subject to stress, frequency and side constraints, so that

$$M(\{X^*\}) = \min_{\{X\}\in F} M(\{X\}) \quad (6)$$

where the feasible domain is specified by the inequalities

$$F = \big\{\{X\}:\sigma_{\mathrm{eff}}(\{X\}) \leq \sigma_{\mathrm{eff}}^a\big\},$$

$$\omega_j(\{X\}) \geq \omega_j^a, \quad j = 1,2, \ldots, k,$$

$$\{X\}_{\min} \leq \{X\} \leq \{X\}_{\max} \quad (7)$$

Situations where several frequencies coalesce and become a multiple frequency may naturally occur. They constitute one of the main difficulties for gradient optimization methods as multiple frequencies are only directionally differentiable. In order to generate correct optimization results, we have to avoid non-differentiability of multiple frequencies in the active set of constraints. In the following sections, we summarize results of design sensitivity analysis of distinct and multiple frequencies, carry through an analytical sensitivity analysis of extremum effective stresses and next the gradient optimization method will be used to solve the optimal design problems with stress and frequency constraints.

2.1. Design sensitivity analysis of distinct frequencies

Let X be one of scalar design parameters. Assuming that λ_j are simple, the frequency sensitivities are obtained from Eq. (2) by direct differentiation