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Sensitivity analysis of the stochastically and periodically forced Brusselator

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Abstract

The problem of sensitivity of nonlinear system limit cycle with respect to small stochastic and periodic disturbances is considered. Sensitivity analysis on the basis of quasipotential function is performed. The quasipotential is used widely in statistical physics (for instance by Graham for analysis of nonequilibrium thermodynamics problem). We consider an application of quasipotential technique to sensitivity problem. For the plane orbit case an approximation of quasipotential is expressed by some scalar function. This function (sensitivity function) is introduced as a base tool of a quantitative description for a system response on the external disturbances. New cycle characteristics (sensitivity factor, parameter of stiffness) are considered. The analysis of the forced Brusselator based on sensitivity function is shown. From this analysis the critical value of Brusselator parameter is found. The dynamics of forced Brusselator for this critical value is investigated. For small stochastic disturbances the burst of response amplitude is shown. For small periodic disturbances the period doubling regime of the transition to chaos scenario is demonstrated.

1. Introduction

Many phenomena of statistical physics (lasers, radio-frequency generators), biological systems and chemical reactions are due to the interaction of couplet oscillations. This is the first mechanical example of "mode locking" that has been observed and studied experimentally at least since the time of Huygens, who discovered "synchronization" of two pendulum clocks mounted on a common wall.

An investigation the periodically forced nonlinear oscillators was started by van der Pol. Subsequent experiments with forced van der Pol and other nonlinear oscillators have discovered the existence of a variety of a periodic, quasiperiodic and aperiodic

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(chaotic) behaviors (see [1,2]). The various transitions ("bifurcations") from periodic to more complicated regimes are a central problem in modern nonlinear dynamic theory. There are many papers devoted to a qualitative analysis of the periodically forced oscillations (see [3,4]). Fundamental results based on mappings analysis of the circumference on to itself were imported by Poincare [5], Denjoy [6] and Arnold [7].

A standard quantification of qualitative understanding of phenomena mentioned above is the bifurcation diagram. Most of the results on bifurcation analysis have been produced for a space of frequency–amplitude parameters of an external periodic force with a fixed nonlinear oscillator. Many researchers have demonstrated a very complicated overlapping structure of entrainment zones (Arnold tongues) of the forced oscillators. Nevertheless, there is obvious interest in the forced result dependence on the parameters of initial system itself. In this paper we are interested in the limit cycle zone of the nonlinear oscillator parameters. It is clear that the response of nonlinear oscillator for the same external periodic force may have an essential difference for different regions of this zone. For some regions the fixed periodic stimulus slightly deforms the initial unforced limit cycle. For other regions it leads to a qualitative change of the oscillation behavior. In these circumstances it would be desirable to have more detailed description of the limit cycle zone of parameters. It needs a quantitative description reflecting the difference in the response of different regions of a cycle sensitivity level to external disturbances. The analytical basis of such description should be some quantitative measure of sensitivity of cycle.

Historically, the Lyapunov exponent was the first value describing the degree of cycle stability. For many systems with auto-oscillations, the mathematical model is the nonlinear deterministic system

$$\dot{x} = f(x) \tag{1.1}$$

with T-periodic solution $x = \xi(t)$ (phase trajectory Γ is a limit cycle). The classical analysis of stability of periodic solutions is based on a linear system of the first approximation

$$\dot{z} = F(t)z, \quad F(t) = \frac{\partial f}{\partial x}(\xi(t))$$

and search of its Lyapunov exponents. As one of the exponents is equal to unity, the given approach in the case of system on a plane tends to a unique quantitative parameter of stability cycle

$$\rho = \mathrm{e}^{\int_0^T trF(t)\,\mathrm{d}t}\,. \tag{1.2}$$

An inequality $\rho < 1$ is the necessary and sufficient condition of an exponential orbital stability. The value ρ specifies (asymptotically) a stability degree of cycles for an initial moment single disturbance. In most cases, the real systems are forced by various disturbances during the motion time. Under these circumstances, ρ is too rough an integrated characteristic to allow to distinguish and compare the stability degrees of various orbit pieces with nonvanishing disturbances.