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Computation of stress intensity factors of interface cracks based on interaction energy release rates and BEM sensitivity analysis

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Abstract

Stress intensity factors of bimaterial interface cracks are evaluated based on the interaction energy release rates. The interaction energy release rate is defined based on the energy release rates of a cracked body, corresponding to two independent loading conditions, actual field and an auxiliary field, and is related to the sensitivities of the potential energies for crack extensions. The potential energy of a cracked body is expressed with a domain integral, which is converted to a boundary integral expression by applying the divergence theorem. By differentiating this expression with the crack length, a boundary integral expression for the interaction energy release rate is obtained. The boundary integral representation for the interaction energy release rate involves the displacement, the traction, and their sensitivity coefficients with respect to the crack length. The boundary element sensitivity analyses are used to calculate these quantities accurately. A regularized boundary integral equation relating the boundary displacement and traction is differentiated with respect to an arbitrary shape parameter to derive the regularized boundary integral equation for the sensitivity coefficients of the boundary displacement and traction. The proposed approach is applied to several cracks in dissimilar media and the results are compared with those obtained by the conventional approach based on the extrapolation method. The analytical displacement and stress solutions for an interface crack between two infinite dissimilar media subjected to uniform stresses at infinity are used to give the auxiliary field, in which the values of the stress intensity factors are known. It is demonstrated that the present method can give accurate results for the stress intensity factors of various bimaterial interface cracks under coarse mesh discretizations.

Keywords: Stress intensity factor; Interface crack; Interaction energy release rate; Boundary element method; Sensitivity analysis

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1. Introduction

Stress intensity factors are important parameters in the investigations of fracture toughness of composite materials, adhesive structures, electronic devices, etc. Various methods for evaluating the stress intensity factors of bimaterial interface cracks have been developed so far [1–9]. Most of the previous works used the boundary element method (BEM) for the elastiostatic analyses of cracked dissimilar materials, because it can give us fairly accurate numerical solutions with discretizing only the boundary. Although the extrapolation method [2,3] seems to be most convenient to determine the stress intensity factors from BEM solutions, a very fine mesh is required in BEM analyses to obtain accurate results. Yuuki and Cho [2] employed Hetenyi's fundamental solution [10,11], which satisfies the interface boundary condition rigorously, as the kernel function of the boundary integral equation used in BEM [12], so that the discretization of the interface boundary was avoided and the accuracy of the numerical results was increased. The method proposed by Tan and Gao [4] and Gao and Tan [5] uses only the direct BEM solutions for the displacements or the tractions at the nodes belonging to the elements adjacent to the crack tip to calculate the absolute value of the stress intensity factor $K_0 = |K_1 + iK_2|$ and the phase angle $\psi = \tan^{-1}(K_2/K_1)$. In their methods, the stress intensity factors are directly related to the displacements and the tractions at the nodes of the quarter point singular elements. Besides, energy methods based on BEM analyses have also been developed [6–9] like in FEM. Miyazaki et al. [6] developed methods which combine BEM and FEM to use virtual crack extension method. They also developed a method [7] to use M_1-integral derived from J-integral. M_1-integral is a contour integral related to the displacement gradients and the stress components. They defined the integration path around the crack tip and calculated the displacement gradients and the stresses as the internal quantities by BEM. de Paula and Aliabadi [8] also used J-integral to evaluate stress intensity factors of interface cracks in orthotropic materials. Xiao and Hui [9] evaluated the energy release rate by means of crack closure method which uses the nodal displacements and tractions of the elements adjacent to the crack tip. The phase angle ψ of the complex stress intensity factor is then determined using the BEM results for the crack opening displacements. Among the previous works, the techniques using virtual crack extension method and M_1-integral method proposed by Miyazaki et al. [6,7] seem to give most accurate results with comparatively coarse meshes.

In this work, we present a method for evaluating stress intensity factors of bimaterial interface cracks in isotropic dissimilar media, based on the interaction energy release rates proposed by Shih and Asaro [13]. The interaction energy release rate is actually equivalent to the M_1-integral defined by the J-integral approach. Shih and Asaro [13] used a domain integral obtained by converting the corresponding M_1-integral representation to evaluate the interaction energy release rates. By using interaction energy release rates, we can evaluate the real and imaginary parts of the complex stress intensity factor separately, so that we can also obtain the phase angle [14,15], which has an important role in characterizing fracture.

The energy release rate is also related to the sensitivity of the potential energy with respect to a crack extension. The virtual crack extension method based on the FEM uses this principle. The potential energy can be converted to a boundary integral, hence, the energy release rate can also be expressed as a boundary integral related to the boundary displacements and tractions, and their sensitivity coefficients with respect to the crack length. Keum and Kwak [16,17] and Bonnet and Xiao [18] utilized this relation to evaluate the energy release rate for cracks in isotropic homogeneous media. They used boundary element sensitivity analyses [19,20] to evaluate the sensitivity coefficients of the displacements and the tractions over the boundary.

In this paper, we derive a boundary integral expression for the interaction energy release rate by differentiating the potential energy for the total field, which is the superposition of the actual field and an auxiliary field, with respect to the crack length. The resulting boundary integral representation