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Criticality and sensitivity analysis of the components of a system

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Abstract

In the design of complex systems there is a great interest to know the relative importance of each of their elements. In this paper, we define a new method for measuring the relative importance of each element of the system. We have to specify that this paper concerns only non-repairable systems and components. We present a way of calculating the criticality of each component for a complex system no matter what the random distribution of the life of the component is. The paper also demonstrates a simple way of calculating how the system life improves when the life of a component is improved.

Keywords: Criticality; Reliability of systems; Reliability importance of components

1. Introduction

The relative importance of the components of a system has been widely studied. The measures of importance are basically of two kinds: functional importance (that is, importance in relation to the reliability of the system) and structural importance. We consider only non-repairable systems and components. Barlow [1], Boland [2] and Tong [3] have studied the structural importance of the components of a system and more recently Meng [4,5] has presented new measures of that kind. Natvig [6] and Bergman [7] propose other measures of the importance of the components of a system. It is also interesting to consult Wall [8], Cheok [9] and Dutuit [10] about measures of importance and some others factors of importance.

In this paper a new method is proposed for the importance of components from a functional point of view by studying how the system life improves when the mean life of a component is improved. With this knowledge one can highlight which component (or components) must be given greater attention, when all the components are independent from each other.

2. Sensitivity measure

The importance of the k-component for the life of the system is calculated by studying the consequence for the mean life of the system by increasing the mean life of a component in Δm_k. This calculation of $(\partial m/\partial m_k)$, gives a measure of the system sensitivity in relation to the k-component.

The calculation of $(\partial m/\partial m_k)$ is generally impossible to do analytically. A simulation method can be used to calculate the mean lives of the system for two values of the mean life of a k-component (for example, m_k and $m_k + \Delta m_k$). When Δm_k is small enough, we obtain the partial derivation as a forward finite-difference equation

$$\frac{\partial m}{\partial m_k} \simeq \frac{\Delta m}{\Delta m_k} \tag{1}$$

This means doing two runs of the simulation per component, but one run of any couple of runs may be the same for all the components. Thus it will be necessary to do $n_v + 1$ runs (n_v number of nodes), where in each run we generate, a large number of random lives of each component, for example, for each component we can generate 100,000 random lives, using a Monte Carlo method. For series and parallel systems when the variables are exponential probability distribution function (PDF) variables, this paper demonstrates that

$$\frac{\partial m}{\partial m_k} = \frac{p'_k m'_k}{m_k} \tag{2}$$

where p'_k is the probability of criticality and can be estimated as the number of times the system life matches the lifetime of the specified k-component divided by the number of simulations done in a run of the programme. m'_k is the critical mean value of the k-component and can be estimated as the sum of the lives of the k-component when it matches the system life/number of times there has been a match between a component life and a system life. m_k is the

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Nomenclature

x_i	Life of the i-component
T	Life of the system
m	Mean value of the life of the system
m_k	Mean value of the k-component life
$\partial m/\partial m_k$	Sensitivity measure or partial variation for the k-component
$\Delta m/\Delta m_k$	Approximation to the theoretical value of $\partial m/\partial m_k$
λ_k	Failure rate of the k-component
p'_k	Probability of criticality for the k-component
m'_k	Critical mean life value for the k-component
n_v	Number of nodes of the system

mean value of the k-component and it can be estimated as the sum of the lives of k-component/number of simulations done in a run of the programme.

Consequently, the method proposed in this paper will only need one run of simulation to calculate the measure of the system sensitivity ($\partial m/\partial m_k$) as the calculation of the parameters p_k, m'_k, and m_k requires only one run assuming that in this run, we generate for example 100,000 random lives of each component, saving much time in order to obtain the same results. For mixed networks, the same formula (2) will be used and compared to the theoretical results for a simple system since the theoretical demonstration is impracticable in more complex systems.

3. Theoretical calculation of sensitivity measure for series and parallel systems

A series and a parallel network can be considered as extreme configurations. Thus the system life for any system of n_v components is,

$$\min_{1\le i\le n_v}(x_i) \le T \le \max_{1\le i\le n_v}(x_i) \tag{3}$$

the expressions at the left and right being the lives for series and parallel systems, respectively.

The general propositions valid for both systems, series and parallel, will be applicable for any other mixed system. Consequently, a demonstration of formula (2) for a series system and a parallel system is presented.

3.1. Series

In this case, the system reliability function is

$$P(T > t) = \prod_{i=1}^{n_v} P(x_i > t) = \mathrm{e}^{-t\sum_{i=1}^{n_v}\lambda_i} \tag{4}$$

and so

$$F_T(t) = 1 - \mathrm{e}^{-t\sum_{i=1}^{n_v}\lambda_i} \tag{5}$$

Therefore, T follows an exponential distribution with a failure rate $\sum_{i=1}^{n_v}\lambda_i$, and the mean life of the system is,

$$m = \frac{1}{\sum_{i=1}^{n_v}\lambda_i} \tag{6}$$

Deriving the formula (6) with respect to m_k, we obtain for series systems,

$$\frac{\partial m}{\partial m_k} = \frac{\lambda_k^2}{\left(\sum_{i=1}^{n_v}\lambda_i\right)^2} \tag{7}$$

3.2. Parallel

In this case, the distribution function of the system life is,

$$P(T \le y) = \prod_{i=1}^{n_v} P(X_i \le y) = \prod_{i=1}^{n_v}(1 - \mathrm{e}^{-y\lambda_i}) \tag{8}$$

from which we obtain the mean life of the system,

$$m = \sum_{i=1}^{n_v}\frac{1}{\lambda_i} - \sum_{ij(i<j)}^{n_v}\frac{1}{\lambda_i+\lambda_j} + \sum_{ijk(i<j)(j<k)}^{n_v}\frac{1}{\lambda_i+\lambda_j+\lambda_k} - \cdots(-1)^{n_v+1}\frac{1}{\lambda_i+\lambda_j+\cdots\lambda_{n_v}} \tag{9}$$

Deriving the formula (9) with respect to m_k, we obtain for parallel systems,

$$\frac{\partial m}{\partial m_k} = \lambda_k^2\left(\frac{1}{\lambda_k^2} - \sum_{i=1(i\ne k)}^{n_v}\frac{1}{(\lambda_i+\lambda_k)^2} + \sum_{ij(i\ne k)(j\ne k)}^{n_v}\frac{1}{(\lambda_i+\lambda_j+\lambda_k)^2} - \cdots(-1)^{n_v+1}\frac{1}{(\lambda_i+\lambda_j+\cdots+\lambda_{n_v})^2}\right) \tag{10}$$

4. Critical component: probability of criticality

For any kind of system, parallel, serial or a mixture, when the systems fail, the life of the system will be the same as that of a component of the system, identifying this component as the critical one, because it is the reason for the failure of the system. The probability that a k component is critical will be the probability of criticality of k, p'_k.