

# Design sensitivity analysis with hypersingular boundary elements

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## Abstract

The finite difference load method for shape design sensitivity analysis requires the calculation of stress and stress gradient on the boundary. In the standard boundary element method, the basic state variables—displacement and traction are continuous, and are considered as very accurate. However, the boundary stress and stress gradient, derived from the differentiation of the state variables and Hooke's law, are discontinuous and have relatively lower accuracy than the basic state variables. The hypersingular boundary integral equation is introduced in this paper to determine the stress and stress gradient in the design sensitivity analysis. The numerical examples demonstrate the accuracy of the design sensitivity using the hypersingular boundary elements. © 2000 Elsevier Science Ltd. All rights reserved.

*Keywords:* Design sensitivity; Hypersingular integral; Stress gradient

## 1. Introduction

As a crucial factor in solving shape optimization problems, the issue of design sensitivity analyses has been extensively studied. The many different approaches to design sensitivity calculation can be grouped into three categories: the finite difference method (FDM), the adjoint structure method (ASM) and the direct differentiation method (DDM). The FDM employs the finite difference formulation to approximate the differentiation of the design variable by solving two boundary value problems [1–3]. The FDM is easy in concept and implementation, but with two serious drawbacks: (a) the accuracy often depends on the perturbation step; (b) the computation cost is relatively higher as the matrices of the perturbed geometry are required. The ASM uses the concept of the material derivative to derive the total change of the structural response with respect to the change of the design variables. By defining an adjoint problem, the final shape design sensitivity can be expressed as an integral of the solutions of the initial problem and the adjoint problem [4–6]. One difficulty related to the ASM is the modeling of the adjoint problem because of the appearance of the concentrated adjoint load, which often results in a poor solution near the adjoint load, thus decreases the accuracy of design sensitivity. The DDM derives the sensitivity by direct differentiation of the bound-

ary integral equations either before or after the boundary discretization [7,8]. The main difficulty of employing the DDM comes from the differentiation of boundary element matrices which involves the differentiation of the boundary variables (displacement and traction), the boundary geometry, as well as the fundamental solutions.

Many attempts have been made to overcome some of the difficulties in evaluating design sensitivities, such as using new boundary element formulations or employing new design variables [9–11]. A finite difference based approach, named finite difference load method (FDLM), has been developed by the author [12], which does not depend on the perturbation step. By analyzing the perturbation procedure of the FDM, the variation of the state variables between the initial geometry and the perturbed geometry can be replaced by a set of perturbation loads which can be obtained from the stress field of the initial problem and the design boundary perturbation. However, the determination of the perturbation loads requires the evaluation of boundary stress and stress gradient. The standard BEM usually provides very accurate displacements and boundary stresses, but not the stress gradient.

In this paper, the hypersingular integral boundary element is used to improve the accuracy of the boundary stress and stress gradient, thus to improve the accuracy of design sensitivities. The basic formulation of the FDLM and the hypersingular boundary element are presented and discussed. Three examples are used to demonstrate the improved accuracy of the design sensitivity calculation.

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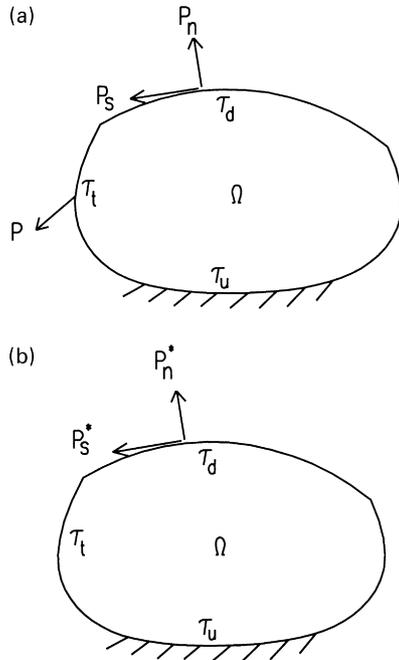


Fig. 1. (a) Initial boundary value problem. (b) Perturbation boundary value problem.

## 2. The finite difference load method

This section briefly presents the basic concept and formulation of the FDLM, full detail can be found in Ref. [12].

Consider a two-dimensional (2D) structure (Fig. 1a) with the domain  $\Omega$  bounded by boundaries  $\tau_u$ ,  $\tau_t$ , and  $\tau_d$ , where  $\tau_u$  is the kinematical boundary,  $\tau_t$  the traction boundary under load  $\mathbf{p}$ ,  $\tau_d$  the traction design boundary with traction components of  $T_s$  and  $T_n$  in tangential and normal directions, respectively. This is the initial boundary value problem.

Given a design boundary perturbation  $\delta \mathbf{r}$  over the design boundary  $\tau_d$ , a new boundary will be formed, noted as  $\tau_1$ . The components of the design boundary perturbation due to the change of the design variable  $\delta \mathbf{r}$  can be expressed as

$$V_n = f_n(\xi) \delta r \quad V_s = f_s(\xi) \delta r \quad (1)$$

where  $V_n$  and  $V_s$  denote the tangential and normal movements of the design boundary due to the perturbation of design variable  $\mathbf{r}$ ;  $f_n(\xi)$  and  $f_s(\xi)$  are the interpolation functions under the local coordinate  $\xi$ . The forms of the interpolation functions depend on the design boundary modeling, such as a linear form for the linear boundary representation, a cubic form for the cubic spline representation.

As shown in Ref. [12], the shape design sensitivity of the structure with respect to the design perturbation  $\delta \mathbf{r}$  is the solution of the same structure under the perturbation load as shown in Fig. 1b, under the same kinematical boundary

condition. The perturbation loads can be derived as

$$p_n^* = \frac{\partial \sigma_n}{\partial n^-} f_n(\xi) - \frac{\partial T_n}{\partial s} f_s(\xi) - 2T_s c_1 + \frac{\partial T_n}{\partial b} \quad (2)$$

$$p_s^* = \frac{\partial \sigma_{sn}}{\partial n^-} f_n(\xi) - \frac{\partial T_s}{\partial s} f_s(\xi) - (\sigma_s - T_n) c_1 + \frac{\partial T_s}{\partial b}$$

where  $p_n^*$  and  $p_s^*$  are the perturbation loads on the design boundary in the normal and tangential directions, respectively,  $\sigma_s$  and  $\sigma_{sn}$  are boundary tangential stress and shear stress, respectively,  $\partial T_n / \partial b$  and  $\partial T_s / \partial b$  are the change rates of the external loads on the design boundary with respect to the design variable  $b$ , and  $c_1 = \lim_{\delta \rightarrow 0} \sin \theta_n / \delta b$  which depends on the design boundary modeling;  $n^-$  presents the inward normal direction,  $T_n$  and  $T_s$  are the boundary traction in the normal and tangential directions, respectively.

It is clear from Eq. (2) that the stress gradients are needed to determine the perturbation loads. By considering the equilibrium equation of a small block along the boundary, these two terms can be obtained by boundary information only. The stress gradients, in polar coordinates, are given as follows:

$$\frac{\partial \sigma_n}{\partial n^-} = k \left( \frac{\partial \sigma_{r\theta}}{r \partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} + X_r \right) \quad (3)$$

$$\frac{\partial \sigma_{sn}}{\partial n^-} = k \left( \frac{\partial \sigma_\theta}{r \partial \theta} + \frac{2}{r} \sigma_{r\theta} + X_s \right)$$

where  $\sigma_{r\theta}$ ,  $\sigma_r$  and  $\sigma_\theta$  are the tangential boundary traction, normal boundary traction and tangential stress, respectively,  $k = 1$  if  $r^+$  is in the  $n^+$  direction, otherwise  $k = -1$ ;  $X_r$  and  $X_s$  are the components of the body force in radial and tangential direction, respectively.

The displacement and stress solutions of the perturbation problem provide the displacement and stress sensitivities at a fixed point. If the constraint is defined on the moving design boundary, the constraint area will move as the design boundary moves. For such cases, the design sensitivity at a point can be expressed by two parts as

$$\frac{dg}{db} = \lim_{\delta b \rightarrow 0} \frac{1}{\delta b} (\delta g_t + \delta g_x) = \frac{\partial g_t}{\partial b} + \frac{\partial g_x}{\partial b} \quad (4)$$

where subscripts  $t$  and  $x$  indicate time (i.e. fixed point) and space, respectively,  $dg/db$  is the final design sensitivity,  $\partial g_t / \partial b$  is the design sensitivity at a fixed area which can be obtained from the solutions of the perturbation problem.  $\partial g_x / \partial b$  can be evaluated as

$$\frac{\partial g_x}{\partial b} = g_{,s} f_s(\xi) + g_{,n} f_n(\xi) \quad (5)$$

where  $s_b$  and  $n_b$  are the components of unit vector  $\delta \mathbf{b} / |\delta \mathbf{b}|$  in tangential and normal directions, respectively.

The numerical implementation of the FDLM for design sensitivity calculation can be summarized as following:

1. Solve the initial boundary value problem (Fig. 1a) under the applied loads using the BEM, i.e.  $\mathbf{Ax} = \mathbf{By}$ .

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