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Shape design sensitivity analysis for fracture conditions

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Abstract

This paper deals with the shape design sensitivity analysis for quasi-brittle plane bodies and implementation of this analysis in numerical method. Special attention is devoted to basic relations of sensitivity analysis which are derived with the help of domain representation of the path-independent *J*-integral and introduction of adjoint system. Numerical technique for finding the sensitivity of *J*-integral with respect to a wide class of the boundary variations and specifically with respect to improved variations is worked out. Important aspects of shape design sensitivity analysis realization related with finite-element modelling, mesh adaptation and smoothing technique are considered. Numerical results of design sensitivity computations, performed for cracked plates loaded by in-plane forces, are presented. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

A lot of papers have been devoted to the shape design sensitivity analysis (SDSA) and optimization problems with integral and local functionals (see for example [1,2]). Expressions for shape design sensitivity have been expressed as boundary integrals or as domain integrals and evaluated using finite element analysis and boundary-element analysis.

Fewer studies have been devoted to the special class of problems for finding relations connecting strengths characteristics with variations of geometrical parameters of the considered bodies. Effective relations connecting the stress intensity factors and the energy release rate with the variations of geometric and material parameters of quasi-brittle bodies and their implementation in FEM are of significant interest in fracture mechanics and in the theory of optimal structural design. These relations give us the possibility to evaluate the sensitivity of fracture criterion with respect to imperfect structural shape and nonideal manufacture and to estimate structural parameters influence of crack propagation. Application of these relations is useful for determination of structural solutions which increase the strength and help to solve the problems of optimal design.

SDSA gives also important relevant information. Note the paper [3], where the growth of an initial crack was treated as a change in shape and it was shown how SDSA leads to a well known expression of Rice's path-independent integral and how this integral can be computed through domain integration, which gives much more accurate results when using FEM (see also [4]).

This paper deals with the shape design sensitivity analysis for quasi-brittle elastic plates. We apply the approach, based on using *J*-integral and on introduction of adjoint system, to derive the design sensitivity analysis relations. Special attention is devoted to basic relations of sensitivity analysis which are derived with the help of well known domain representation of the *J*-integral.

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2. Sensitivity analysis with respect to shape variations

Consider a deformed elastic body occupying a two-dimensional region Ω with boundary $\mathbf{\Gamma}$. Surface tractions $\mathbf{T}$ are given on the boundary contour $\mathbf{\Gamma}_\sigma$ while zero displacements are assigned to the $\mathbf{\Gamma}_u(\mathbf{\Gamma}=\mathbf{\Gamma}_\sigma+\mathbf{\Gamma}_u)$. The body contains a crack modelled by a rectilinear notch $\mathbf{\Gamma}_f$. The notch is traction free. The state of a stressed and deformed body is described by the following boundary value problem of the theory of elasticity

$$\nabla\boldsymbol{\sigma}=0,\quad \boldsymbol{\sigma}=\mathbf{C}\boldsymbol{\epsilon},\quad \boldsymbol{\epsilon}=\frac{1}{2}(\nabla\mathbf{u}+(\nabla\mathbf{u})^T),$$
$$(\boldsymbol{\sigma}\mathbf{n})_{\Gamma_\sigma}=\mathbf{T},\quad (\mathbf{u})_{\Gamma_u}=0,\quad (\boldsymbol{\sigma}\mathbf{n})_{\Gamma_f}=0 \tag{1}$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\epsilon}$ and $\mathbf{u}$ are respectively, the stress tensor, the strain tensor and the displacement vector, and by means of $\mathbf{C}$ and $\mathbf{n}$ in Eq. (1) we denote the elastic modulus tensor and the unit vector pointing in the direction of an outward normal to the boundary of the body $\mathbf{\Gamma}$ and to the boundary of the crack $\mathbf{\Gamma}_f$, while $\mathbf{T}$ is a given vector function of the space coordinates. The basic relations (1) are derived under the assumptions that the laws of static are obeyed and the strains are small.

In the frame of the plane theory of elasticity the brittle fracture criteria can be written in the form $J\leq G_C$, where G_C is the given brittle strength constant and J is the path-independent integral. For our purposes it is important that the contour integral can be transformed [4] to the domain integral expression (see Fig. 1).

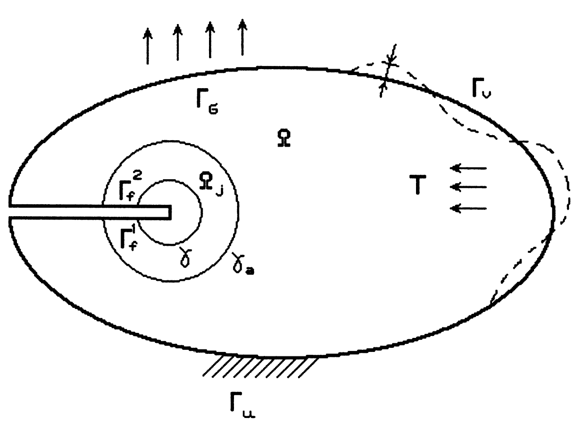

Fig. 1. Cracked body with varied boundaries and domain decomposition for the evaluation of J-integral.

$$J=\int_\gamma\left[W(\mathbf{nl})-(\boldsymbol{\sigma}\mathbf{n})\frac{\partial\mathbf{u}}{\partial\mathbf{l}}\right]\mathrm{d}\gamma$$
$$=\int_{\Omega_J}\left[\frac{\partial\mathbf{u}}{\partial\mathbf{l}}\boldsymbol{\sigma}-W\mathbf{l}\right]\nabla\psi\mathrm{d}\Omega_J \tag{2}$$

Here γ is any path beginning at the bottom crack face and ending on the top face as shown in Fig. 1, $\mathbf{n}$ is the outward normal to γ, dγ is the increment of arc length along γ, $\mathbf{l}$ is the unit vector tangential to the crack, $\Omega=\Omega_I+\Omega_J+\Omega_E$, ψ is a sufficiently smooth function in Ω_J that is unity on γ and vanishes on γ_a: $(\psi)_\gamma=1$, $(\psi)_{\gamma_\mathrm{a}}=0$, and Ω_J is a simply connected region enclosed by the contour $\gamma+\gamma_\mathrm{a}+\mathbf{\Gamma}_f^1+\mathbf{\Gamma}_f^2$. The energy release rate domain integral (2) is useful for modelling of quasi-brittle fracture conditions and give the possibility to diminish the loss of accuracy from numerical evaluation of the J integral. The domain integral expression (2) is also convenient for deriving design sensitivity analysis relations and will be used for this purpose in our investigations.

To evaluate the influence of shape variation on the value of the J-integral we perform the variation of the part $\mathbf{\Gamma}_v$ of the boundary $\mathbf{\Gamma}$ admitting the normal boundary variations $\delta t(P)$ for the boundary points $P\in\Gamma_v$ (see Fig. 1). As a result we will obtain the increments $\delta\mathbf{u}$, $\delta\boldsymbol{\epsilon}=1/2(\nabla\delta\mathbf{u}+(\nabla\delta\mathbf{u})^\mathrm{T})$, $\delta\boldsymbol{\sigma}$ for displacements, strains and stresses. Let us derive the variation δJ with respect to the fields variations $\delta\mathbf{u}$, $\delta\boldsymbol{\sigma}$. For this purpose we vary the domain integral representation for the energy release rate

$$\delta J=\int_{\Omega_J}\left[\left(\frac{\partial\mathbf{u}}{\partial\mathbf{l}}\right)\delta\boldsymbol{\sigma}\nabla\psi+\left(\frac{\partial\delta\mathbf{u}}{\partial\mathbf{l}}\right)\boldsymbol{\sigma}\nabla\psi-(\boldsymbol{\sigma}\delta\boldsymbol{\epsilon})\frac{\partial\psi}{\partial\mathbf{l}}\right]\mathrm{d}\Omega_J$$
$$=\int_{\Omega_J}\left[\left(\frac{\partial\mathbf{u}}{\partial\mathbf{l}}\right)\delta\boldsymbol{\sigma}\nabla\psi+\nabla\left[(\nabla\psi\boldsymbol{\sigma}\delta\mathbf{u})\mathbf{l}-(\boldsymbol{\sigma}\delta\mathbf{u})\frac{\partial\psi}{\partial\mathbf{l}}\right]-\delta\mathbf{u}\left(\frac{\partial\boldsymbol{\sigma}}{\partial\mathbf{l}}\right)\nabla\psi\right]\mathrm{d}\Omega_J \tag{3}$$

Using index notation for tensor values, the expression for δJ can be rewritten as

$$\delta J=\int_{\Omega_J}\left[\frac{\partial\psi}{\partial x_j}\frac{\partial u_i}{\partial x_k}l_k\delta\sigma_{ij}+\frac{\partial}{\partial x_k}\left(\frac{\partial\psi}{\partial x_j}\sigma_{ij}l_k\delta u_i-\frac{\partial\psi}{\partial x_j}l_j\sigma_{ik}\delta u_i\right)-\frac{\partial\psi}{\partial x_j}l_k\frac{\partial\sigma_{ij}}{\partial x_k}\delta u_i\right]\mathrm{d}\Omega_J$$

To eliminate the derivatives of the displacements variations with respect to space coordinates let us transform the integral of the expression in square brackets