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Design sensitivity analysis and shape optimization of structural components with hyperelastic material

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Abstract

A continuum-based design sensitivity analysis (DSA) method is developed for structural components with hyperelastic (incompressible) material. A mixed variational principle (MVP) and the total Lagrangian formulation are used for nonlinear analysis. Effects of large displacements, large strains, and material nonlinearities are included in the analysis model, using appropriate kinematics and constitutive relations. The material property and shape DSA using both the direct differentiation method (DDM) and the adjoint variable method (AVM) are discussed. For shape DSA, the material derivative concept is used to compute effects of the shape variation. The boundary displacement and isoparametric mapping methods are employed to compute the design velocity field. Both hydrostatic pressure and structural stiffness are considered as constraints for design optimization, which is carried out by integrating shape design parameterization, design velocity computation, DSA, nonlinear analysis, and the optimization method. Examples such as, an engine mount and a bushing demonstrate the feasibility of the proposed optimization method for designing structural components using hyperelastic material.

1. Introduction

Hyperelastic material, such as rubber and rubber-like material, is very versatile and adaptable, and has long been used successfully in numerous engineering applications. Rubber possesses inherent damping, which is particularly beneficial when a resonant vibration is encountered, and it can store more elastic energy than steel. For hyperelastic material, the bulk modulus, which is associated with the volume change of the structural component, is much larger than the shear modulus, which is associated with the shape deformation of the structural component.

These properties make the hyperelastic material conducive to a wide range of applications in modern industry, such as weather-stripping for insulation, and bushings and engine mounts for noise, vibration, and harshness (NVH) control. To obtain a meaningful shape optimal design of a hyperelastic solid, it is necessary to accurately describe the material properties, thoroughly understand the nonlinear structural analysis procedure, and to correctly formulate the structural design optimization problem. Many works have been published in the first two areas. For design optimization, DSA, which deals with the effect of change of design variables on the structural response, plays an important role. Various DSA methods for linear structures with sizing and shape design variables have been developed and are well documented [1–3]. For nonlinear structures, Choi and Santos [4], Santos and Choi [5,6], and Choi [7] developed sizing and

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shape DSA methods using the continuum approach. The method was extended to handle geometric nonlinear analysis with linear incompressible material [8]. Using the control volume concept, Tortorelli [9] developed a DSA method for nonlinear structures with incompressible material.

It was shown by Gent that a failure occurs when a structural component with the hyperelastic material comes under a certain hydrostatic tensile stress [10]. On the other hand, the stiffness characteristic is an important design consideration in shape optimal design of the engine mount or bushing for vibration isolation [11]. That is, to reduce the NVH of a vehicle system [12,13], the optimum design can be obtained using a two-level approach. First, the system level DSA and optimization can be carried out to find optimum gages and topology of the vehicle body, and optimum stiffness and damping characteristics of engine mounts and bushings. In this design formulation, the weight can be considered as the cost, whereas the NVH are treated as design constraints. Once optimum stiffness and damping characteristics of engine mounts and bushings are determined from the system level design, then shapes of engine mounts and bushings can be optimized to yield these stiffness and damping characteristics.

The objective of this paper is to develop material property and shape DSA and optimization methods using the continuum approach for hyperelastic structural components. The Mooney–Rivlin energy density function is employed to describe the material property. The FEA code ABAQUS [14] that utilizes the MVP is used for nonlinear structural analysis. Numerical examples are presented to demonstrate the efficiency and accuracy of the proposed method.

For the optimization procedure, the data of shape design parameterization, design velocity computation, nonlinear analysis, and shape DSA are integrated. In this paper, a geometric and finite element model generation tool PATRAN [15] and VMA Engineering's design optimization tool DOT [16] are used for design optimization.

2. Governing equations for structures with hyperelastic materials

In this paper, conventional notations [17] of nonlinear structural analysis are used. The motion of a body in a fixed Cartesian coordinate system is shown in Fig. 1. The body may experience large displacements, large rotations, and large strains, with the material nonlinear constraint. The coordinate of a point P on the body is $[{}^0x_1,\ {}^0x_2,\ {}^0x_3]$ at time 0, $[{}^tx_1,\ {}^tx_2,\ {}^tx_3]$ at time t, and $[{}^{t+\Delta t}x_1,\ {}^{t+\Delta t}x_2,\ {}^{t+\Delta t}x_3]$ at time $t+\Delta t$, where the left superscript indicates the configuration time at which the quantity occurs. If the quantity under consideration occurs in the same configuration in which it is also measured, then the left subscript is omitted, e.g., ${}^{t+\Delta t}_{t+\Delta t}z_i \equiv {}^{t+\Delta t}z_i$. For the derivative of a quantity with respect to a variable, the left subscript indicates the configuration in which the variable is measured. For example, for the ith component ${}^tz_i$ of the displacement,

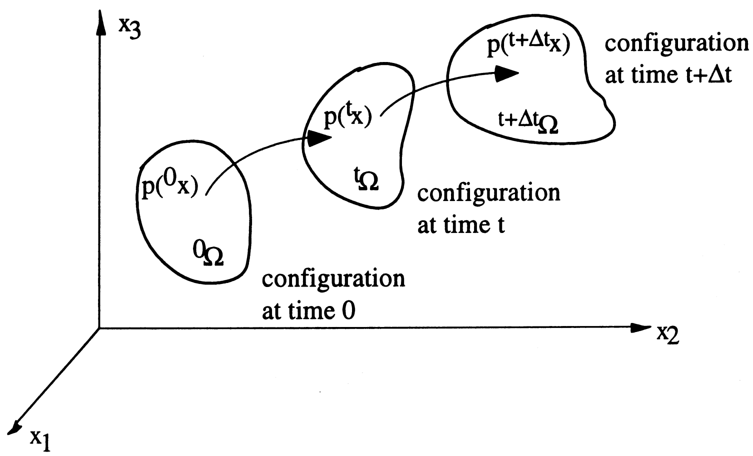

Fig. 1. Motion of the body in fixed cartesian coordinate system.