



Shape sensitivity analysis for energy release rate evaluation and its application to the study of three-dimensional cracked bodies

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Abstract

The energy release rate is an important parameter for the analysis of cracked bodies in linear elastic fracture mechanics. This parameter, usually denoted by G , is equivalent to \dot{I} , the rate of change with respect to crack change of the energy available for fracture. In this paper, crack growth is simulated by an action of change of the shape of the body characterized by an appropriate known smooth velocity field \mathbf{v} defined over the domain of the body. A general (integral) expression for \dot{I} using shape sensitivity analysis based on distributed parameters is also obtained in this paper. Since this expression depends on the displacement field \mathbf{u} and on $\nabla\mathbf{v}$, a simple post-processing technique is required for the numerical evaluation of this expression. An adaptive finite element analysis is performed in order to ensure a good accuracy during the numerical evaluation of \dot{I} . Finally, well known three-dimensional examples in fracture mechanics are considered in order to illustrate the potentiality of the proposed methodology. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

An important concept in the analysis of elastic cracked bodies is the energy release rate. This parameter, denoted by G , is the rate of change with respect to crack advance of the energy available for fracture and it was first introduced by Griffith [5] in his renowned work in which he applies the principle of energy conservation to the study of crack growth.

The basic idea used by Griffith is that the critical state of a crack will occur if the externally added or the internally released energy stored in a body provides the required energy for crack advance. According to the energy balance concept adopted by Griffith, the criteria that he arrived at is a necessary condition for crack advance.

In some simple two-dimensional elastic fracture problems, it is possible to express the potential energy as an explicit function of crack “length”, which enables us to obtain G in a direct form. An example is the problem analyzed by Griffith, an infinite plate containing a straight-through crack subjected to uniform tension. However, the above-mentioned in general is not possible for complex geometries and in particular for three-dimensional cracked bodies, where the characteristic “length” of the fracture should be replaced by parameters (fields) that define the propagation mode of the crack front.

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To solve this difficulty, several procedures both numerically and experimentally have been developed in fracture mechanics. Later, in articles by Sanders [14], Cherepanov [3] and Rice [13], the path-independent integral J was introduced as a new global parameter of fracture mechanics. This integral is usually interpreted as an energy flux through the cylindrical surface enclosing the crack front. Since J is path-independent, it can be determined in the easiest way by selecting a path along which the integration can be carried out conveniently. In this way, the integral provides a relatively simple alternative to determine the energy release rate in the elastic case and is also frequently used in the case of the initiation of crack propagation in plastic bodies.

More recently, with the advent of computers and the generalization of approximation methods in stress analysis, fracture mechanics problems began to be approached by numerical methods [11]. Among them, a numerical method widely used in the literature consists of the employment of a finite difference scheme to evaluate G starting from two calculations of the potential energy associated with different crack sizes. In this case, each mode of crack growth requires an additional complete finite element analysis, including the mesh generation and boundary condition specification. Subsequently, the release energy rate in the finite element model is evaluated when a crack is extended by a small amount controlled by coordinates of the position of the crack tip. Named Virtual Crack Extension Methods, this approach has been first developed by Hellen [8] and Parks [12] in discrete models. In a somewhat more general form, Sussman and Bathe [15], using an isoparametric finite element mesh, derive the potential energy with respect to perturbations at nodal point coordinates. This formulation has been used to compute fracture parameters and improve the initial mesh using a mesh optimization technique.

A similar procedure was incorporated by Zumwalt and El-Sayed [19] in sensitivity analysis directly calculating the derivative of the finite element stiffness matrix in the equilibrium equation. Simultaneously, De Lorenzi [9] developed analytical expressions for the energy release rate using virtual crack extension methods. For certain restrictions on crack geometry, he shows that the energy release rate reduces to the three-dimensional form of the J integral.

A different alternative to the previous ones is the use of shape sensitivity analysis, which allows to directly obtain an expression of this derivative (and derivatives of greater order) although the potential energy as an explicit function of the crack “length” is not available. In this paper, we apply this approach as a systematic methodology to directly obtain the expression of potential energy release rate in three-dimensional cracked bodies. As we shall see later, in this procedure the crack growth is interpreted as a change of the shape of the body and is characterized by a smooth velocity field \mathbf{v} . Starting from this point and using entirely similar transformations to the existent ones in Continuum Mechanics, it is possible to express the potential energy as an implicit function of the change of shape. The first derivative of this expression can be now easily obtained from shape sensitivity analysis techniques.

With the purpose of making this paper self-contained, first we study the variation of the potential energy in the initial configuration of a body, assuming arbitrary change of the initial shape. Second, the change of the initial shape is properly adopted introducing a velocity field (defined by a weighted average technique) which simulates the crack advance. In this way, we obtain the general expression of the energy release rate as an integral over the initial configuration of a cracked body. Finally, several particular cases of three-dimensional cracked bodies are presented. Using a finite element approximation in order to obtain the strain and stress distribution over the initial cracked configuration and later using a post-processing technique, the energy release rate for appropriate velocity fields at the front of the crack is evaluated by numerical integration. These values are compared with the available results from the literature in order to show the potentiality of the present approach.

2. Shape sensitivity analysis

In the present section, we introduce the concept of shape change and we shall study the behavior of functionals when the shape of a body is modified. Proposed originally by Cèa [2] and widely discussed by Haug et al. [7] this approach simulates a change in shape by a (known) *motion* from an *initial configuration* to a *deformed* configuration.

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