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Sensitivity Analysis for Strongly Nonlinear Quasi-Variational Inclusions

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Abstract—In this paper, we use the implicit resolvent operator technique to study the sensitivity analysis for strongly nonlinear quasi-variational inclusions. Our results improve and generalize some of the recent ones. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords—Nonlinear quasi-variational inclusion, Sensitivity analysis, Resolvent operator, Hilbert space.

1. INTRODUCTION

It is well recognized that variational inequalities play a fundamental role in the current mathematical technology. Thus, in recent years, the theory of classical variational inequalities has been extended and generalized to study a wide class of problems arising in mechanics, physics, optimization and control, nonlinear programming, economics and transportation equilibrium, and engineering sciences, etc. For details, we refer to [1–15] and the references therein. A useful and important generalization of variational inequalities is called the quasi-variational inclusion. Ding [16] and Huang [17,18] have used the implicit resolvent operator techniques to study the existence of quasi-variational inclusions. Sensitivity analysis of solutions for variational inequalities have been studied by many authors using quite different methods. By using the projection technique, Dafermos [19], Mukherjee and Verma [20], Noor [21] and Yen [22] dealt with the sensitivity analysis of solutions for variational inequalities. By using the implicit function approach that makes use of so-called normal mappings, Robinson [23] studied the sensitivity analysis of solutions for variational inequalities. Recently, Noor and Noor [24] dealt with the sensitivity

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analysis of solutions for the quasi-variational inclusions by using the implicit resolvent equations technique without assuming the differentiability of the given data.

Inspired and motivated by recent research works in this field, in this paper, by using the implicit resolvent operator technique, we study the sensitivity analysis for strongly nonlinear quasi-variational inclusions. Our results improve and generalize the known results of [19,24].

2. PRELIMINARIES

Let H be a real Hilbert space endowed with the norm $\|\cdot\|$ and inner product $\langle \cdot, \cdot \rangle$, respectively. Let $N : H \times H \rightarrow H$ be a nonlinear mapping and $M : H \times H \rightarrow 2^H$ be a maximal monotone mapping with respect to the first argument.

We consider the problem of finding $u \in H$ such that

$$0 \in N(u, u) + M(u, u). \quad (2.1)$$

Problem (2.1) is called the generalized strongly nonlinear mixed quasi-variational inclusion.

Now, we give some special cases of problem (2.1) as follows.

- (I) If $M(\cdot, y) = \partial\varphi(\cdot, y)$ for each $y \in H$, where $\varphi : H \times H \rightarrow R \cup \{+\infty\}$ such that, for each fixed $y \in H$, $\varphi(\cdot, y) : H \rightarrow R \cup \{+\infty\}$ is a proper convex lower semicontinuous function on H and $\partial\varphi(\cdot, y)$ denotes the subdifferential of function $\varphi(\cdot, y)$, then problem (2.1) is equivalent to finding $u \in H$ such that

$$\langle N(u, u), v - u \rangle \geq \varphi(u, u) - \varphi(v, u), \quad \forall v \in H,$$

which is called the generalized strongly nonlinear mixed quasi-variational inequality.

- (II) If $N(u, v) = Tu + Sv$ for all $u, v \in H$, where $S, T : H \rightarrow H$ are two nonlinear mappings, then problem (2.1) is equivalent to finding $u \in H$ such that

$$0 \in Tu + Su + M(u, u), \quad (2.2)$$

which is called the generalized nonlinear mixed quasi-variational inclusion.

- (III) If $S = 0$, then problem (2.3) is equivalent to finding $u \in H$ such that

$$0 \in Tu + M(u, u), \quad (2.3)$$

which is called the mixed quasi-variational inclusion (see [24]).

We recall that, if T is a maximal monotone mapping, then the resolvent operator J_ρ^T associated with T is defined by

$$J_\rho^T(u) = (I + \rho T)^{-1}(u), \quad \forall u \in H, \quad (2.4)$$

where $\rho > 0$ is a constant and I denotes the identity mapping. It is well known that $J_\rho^T : H \rightarrow H$ is a single-valued nonexpansive mapping.

REMARK 2.1. Since M is a maximal monotone mapping with respect to the first argument, for any fixed $y \in H$, we define

$$J_\rho^{M(\cdot, y)}(u) = (I + \rho M(\cdot, y))^{-1}(u), \quad \forall u \in H,$$

which is called the implicit resolvent operator associated with $M(\cdot, y)$.

We now consider the parametric version of problem (2.1). To this end, let A be a nonempty open subset of H in which the parametric λ takes values. Let $N : H \times H \times A \rightarrow H$ be a nonlinear mapping and $M : H \times H \times A \rightarrow 2^H$ be a maximal monotone mapping with respect to the first argument. The parametric generalized strongly nonlinear mixed quasi-variational inclusion problem is to find $u \in H$ such that

$$0 \in N(u, u, \lambda) + M(u, u, \lambda). \quad (2.5)$$

We now establish the equivalent between problem (2.5) and the problem of finding fixed point for the nonlinear mapping associated with the implicit resolvent operator, which is the main motivation of our next results.

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