ELSEVIER

Comput. Methods Appl. Mech. Engrg. 189 (2000) 613–624

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Perturbation mapping method for sensitivity analysis of three-dimensional cracks near a free surface

Yonglin Xu, Ahmed K. Noor *

Center for Advanced Computational Technology, University of Virginia, NASA Langley Research Center, Mail Stop 201, Hampton, VA 23681, USA

Received 28 May 1999

Abstract

A perturbation mapping method and a computational procedure are presented for evaluating the sensitivity coefficients of the stress intensity factors for three-dimensional planar cracks near a free surface. The boundary integral equations for evaluating the sensitivity coefficient are solved by using the boundary element method. Each of the geometric parameters that affect the stress intensity factor (such as, crack orientation, distance from the free surface, and crack shape parameters) is given a perturbation which defines a mapping between the original and perturbed coordinate systems, from which the sensitivity coefficients are derived. The sensitivity coefficients obtained by the perturbation mapping method are validated by comparing them with those obtained by the finite difference method. Numerical results for penny-shaped and elliptical cracks are presented showing the variation of the sensitivity coefficients with various geometric and material parameters.

1. Introduction

Due to the importance of stress intensity factors in determining the fatigue life of materials, considerable attention has been devoted to their accurate determination [1–6]. The problem of an arbitrary-shaped crack in an infinite or semi-infinite linear elastic solid can be generally formulated in the form of traction-boundary integral equations [7,8]. Efficient boundary integral equations and finite element methods have been developed for the solution of these equations [9,10]. Despite the progress made to date, the use of numerical fracture mechanics calculations in automated optimum design of structures requires efficient techniques for calculating the sensitivity of the stress intensity factors to variations in crack geometry and material parameters. The sensitivity coefficients (derivatives of the stress intensity factors with respect to geometric and material parameters) can be used to:

1. Assess the effects of uncertainties in the material and geometric parameters on the accuracy of the calculated stress intensity factors.
2. Predict the changes in the stress intensity factors due to changes in the parameters.

Although a number of techniques have been developed for evaluating the sensitivity coefficients of the response quantities of structures, to the authors' knowledge no application of these techniques to the boundary integral equation formulation of fracture mechanics has been reported. The techniques developed for sensitivity analysis can be grouped into three categories: analytical direct differentiation methods, semi-analytical or quasi-analytical methods, and finite difference methods [11–13].

* Corresponding author. Tel.: +1-757-864-1978; fax: +1-757-864-8089.
E-mail address: a.k.noor@larc.nasa.gov (A.K. Noor).

0045-7825/00/$ - see front matter
PII: S0045-7825(99)00314-X

The present study focuses on the sensitivity analysis of crack problems. Specifically, this paper presents a perturbation mapping method and a computational procedure for evaluating the sensitivity coefficients of the stress intensity factors for three-dimensional planar cracks near a free surface. The boundary integral equations are solved by using the boundary element method. To fix ideas, only three-dimensional cracks under mode I loading (opening mode) are considered.

2. Mathematical formulation

2.1. Stress intensity factor

Fig. 1 shows an elliptical crack near a free surface of a linearly elastic isotropic tensile specimen. The stress intensity factor depends on the material parameters as well as on the geometric parameters a, b and the distance of the crack from the free surface t. The boundary integral equation for a crack in the x–y plane can be written as

$$\sigma_{33} = \int_{\Omega} S_{333}(\xi,\eta,x,y)\Delta u_3(\xi,\eta)\,\mathrm{d}\Omega(\xi,\eta), \tag{2.1}$$

where the crack surface is defined by Ω. The difference in the crack surface displacements Δu_3 is represented by $\Delta u_3 = u_3^+ - u_3^-$, and

$$S_{333} = s_{333} + \bar{s}_{333}. \tag{2.2}$$

The first term on the right-hand side of Eq. (2.2) represents Green's function for a crack embedded in an infinite solid, and $\bar{s}_{333}$ is the contribution of the free surface. The expressions of s_{333} and $\bar{s}_{333}$ are given by [6]:

$$s_{333} = \frac{\mu}{4\pi(1-\nu^2)}\frac{1}{r_1^3}, \tag{2.3}$$

$$\bar{s}_{333} = \frac{\mu}{4\pi(1-\nu^2)}\left\{-\frac{5-20\nu+24\nu^2}{r_2^3} + \frac{12(1-2\nu)(1-\nu)}{r_2(r_2+y+\eta)^2} - \frac{6\left[2\nu(1-2\nu)(y+\eta)^2 - 3y\eta\right]}{r_2^5}\right\}. \tag{2.4}$$

In Eqs. (2.3) and (2.4), μ and ν are the shear modulus and Poisson's ratio of the isotropic material. The functions r_1 and r_2 are given by

$$r_1 = \sqrt{(\xi-x)^2 + (\eta-y)^2}, \quad r_2 = \sqrt{(\xi-x)^2 + (\eta+y)^2}. \tag{2.5}$$

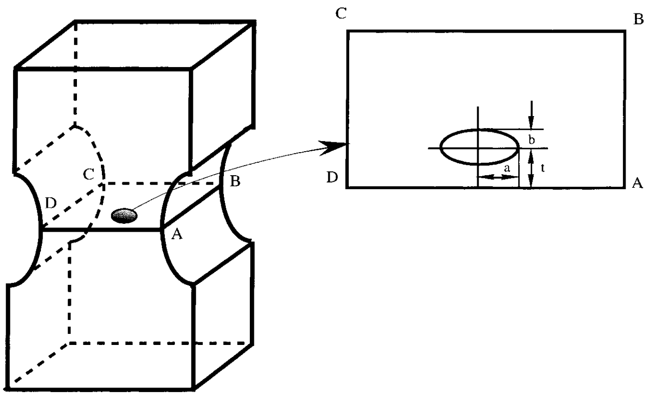

Fig. 1. Three-dimensional planar crack near a free surface.