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Two methods of sensitivity analysis for multibody systems with collisions

M. Schulz[a,*], H. Brauchli[b]

[a]Robert Bosch GmbH, FV/SLT2, P.O. Box 30 02 40, 70442 Stuttgart, Germany
[b]ETH Zentrum, Institut für Mechanik, CH-8092 Zurich, Switzerland

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Abstract

In this paper, two semi-analytical methods of sensitivity analysis, the Direct Differentiation Method and the Adjoint Variable Method, are extended to multibody systems with collisions. Elementary impact theory is used for the description of collisions, i.e. the duration of the impact, configuration changes during the impact, wave propagation and so on are neglected. The variables of state are discontinuous at the time of impact. As a consequence, the design derivatives of the variables of state and hence the adjoint variables are also discontinuous. An algebraic relationship between the adjoint variables before and after the impact is derived. The number of adjoint equations are not increasing with the number of design variables whereas the number of equations in the Direct Differentiation Method depends linearly on the dimension of the design space. This is not only true for the two methods as they are known from the literature but also for the extended methods. Although the implementation of the extended Adjoint Variable Method in a numerical simulation package is rather elaborate and cumbersome, it has been done successfully as is shown with the aid of a circuit breaker mechanism with three contacts. The numerical efficiency of the new approach is discussed in case of this example system. © 2000 Elsevier Science Ltd. All rights reserved.

Notation

Column vectors are denoted by bold lower-case letters and matrices, in general, by bold upper-case letters. The derivative of a scalar function a(b) with respect to a column vector b is

* Corresponding author. Tel.: +49-170-244-8314.
E-mail address: marcus.schulz2@de.bosch.com (M. Schulz).

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denoted by $(\partial a)/(\partial \mathbf{b})$ and is a row vector

$$\frac{\partial a}{\partial \mathbf{b}} = \begin{bmatrix} \frac{\partial a}{\partial b_1} & \frac{\partial a}{\partial b_2} & \cdots \end{bmatrix},$$

where b_1, b_2, ... are the entries of **b**. The derivative $(\partial \mathbf{c})/(\partial \mathbf{b})$ of a column vector $\mathbf{c}(\mathbf{b})$ with respect to **b** is the Jacobian matrix of **c** and defined according to

$$\frac{\partial \mathbf{c}}{\partial \mathbf{b}} = \begin{bmatrix} \frac{\partial c_1}{\partial b_1} & \frac{\partial c_1}{\partial b_2} & \cdots \\ \frac{\partial c_2}{\partial b_1} & \frac{\partial c_2}{\partial b_2} & \\ \cdots & & \end{bmatrix},$$

where c_1, c_2, ... are the entries of **c**. The transposed of a matrix or a column vector is indicated by an upper index T. A dot denotes differentiation with respect to time.

1. Introduction

Sensitivity analysis theory was originally developed in optimal control theory but the Direct Differentiation Method and the Adjoint Variable Method have also extensively been applied in structural mechanics and in multibody system dynamics [1]. Since sensitivity analysis methods provide gradient information that is required by local optimization strategies, they have very often been discussed in the context of design optimization [2]. In the dimensional synthesis of linkages for example, a rather common task is the determination of the geometry of a linkage such that one link performs a prescribed motion. This task is called motion generation and can be formulated as a nonlinear optimization problem introducing a measure for the deviation of the motion of the current linkage from the prescribed motion. This error must be minimized choosing suitable values of the geometry parameters. In this context, sensitivity analysis denotes the computation of the derivatives of the error with respect to the geometry parameters. With the aid of these design derivatives a direction for a redesign can be found.

In the present paper, however, sensitivity analysis is discussed for integral-type performance measures evaluating the transient dynamic behaviour of mechanisms. In [3] it was shown that such performance measures are discontinuous for most technologically relevant mechanisms, if collisions are described by one of the elementary impact theories. For this reason, local strategies are not suited for the optimization of the transient dynamic behaviour of mechanisms with collisions. Global strategies embodying stochastic elements should be applied instead. However, this does not mean that there would be no need for the extension of existing sensitivity analysis methods to systems with collisions. Clustering methods, for example, combine stochastic elements with local optimization procedures can be used for the design of mechanisms with collisions. In [4] the Adjoint Variable Method has been applied to mechanisms with intermittent motion. Collisions have been modelled by inserting a stiff spring between two impacting bodies. The major drawback of this model is a numerical one. The differential equations of motion can become stiff during the impact interval. In this respect, the description of collisions by an elementary impact theory is preferable. Only algebraic equations