



Gaussian and non-Gaussian stochastic sensitivity analysis of discrete structural system

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Abstract

The derivatives of the response of a structural system with respect to the system parameters are termed sensitivities. They play an important role in assessing the effect of uncertainties in the mathematical model of the system and in predicting changes of the response due to changes of the design parameters. In this paper, a time domain approach for evaluating the sensitivity of discrete structural systems to deterministic, as well as to Gaussian or non-Gaussian stochastic input is presented. In particular, in the latter case, the stochastic input has been assumed to be a delta-correlated process and, by using Kronecker algebra extensively, cumulant sensitivities of order higher than two have been obtained by solving sets of algebraic or differential equations for stationary and non-stationary input, respectively. The theoretical background is developed for the general case of multi-degrees-of-freedom (MDOF) primary system with an attached secondary single-degree-of-freedom (SDOF) structure. However, numerical examples for the simple case of an SDOF primary–secondary structure, in order to explore how variations of the system parameters influence the system, are presented. Finally, it should be noted that a study of the optimal placement of the secondary system within the primary one should be conducted on an MDOF structure. © 2000 Civil-Comp Ltd. and Elsevier Science Ltd. All rights reserved.

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1. Introduction

In many cases of engineering interest, it has become quite common to use stochastic processes to model loads. In these cases, the structural response needs to be adequately evaluated in a probabilistic sense by means of cumulants or statistical moments of any order [1], if the closed form expression of the probability density function cannot be determined.

For linear systems excited by normal (or Gaussian) input, the response process is also normal and the moments, or alternatively the cumulants up to the second

order, are able to fully characterise the probability density function of both input and output processes.

In some circumstances, the excitation is significantly non-normal, as in the case of quadratic terms commonly used in fluid mechanics, and the probabilistic characterisation of the response requires higher order moments (or cumulants), which can be evaluated once higher order correlation functions of the input are known [2]. Alternatively, the non-normal input process can be obtained as the solution of a set of first-order linear differential equations subjected to a delta-correlated process [3]. The delta-correlated processes are also known as non-normal white noise processes, or Levy white noise processes [4]. An important class of non-normal processes consists of memoryless non-linear transformations of normal processes [5,6]. As an example, polynomials of Gaussian processes have been

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applied to model the wind and wave forces acting on structural systems [7,8].

Once the input of the system is fully characterised, attention has to be devoted to the assumption of the values of the system parameters. The basic question of stochastic sensitivity is the following: “what is the influence, in some stochastic sense, of the parameters of the system on the solution of this system?” [9]. This problem is important as a real system and its idealised mathematical model cannot be identified exactly. The so-called sensitivities are the derivatives of the response of a structural system with respect to the system parameters such as stiffness, mass, damping, etc. They play an important role in assessing the effect of uncertainties in the mathematical model of the system and in predicting changes in the response due to changes in the design parameters. Indeed, in the case of complex mechanical and structural systems, the evaluation of sensitivities (sensitivity analysis) provides a very powerful method for the choice of optimal structural parameters, rather than through the use of standard optimisation methods.

Many books and papers have been published on the sensitivity analysis of structural systems under dynamic deterministic loads in both time and frequency domain [10–12] and under Gaussian stochastic processes [9,13,14]. Recently, it has also been shown that the sensitivity analysis can play an important role in the design of composite primary–secondary structural systems [15,16]. In particular, this analysis can be used to design the optimal secondary structural parameters in such a way that the primary substructure response is reduced.

In this paper, a formulation for finding the stochastic sensitivities of a discrete structural system subjected to stochastic Gaussian or non-Gaussian input is presented. The proposed formulation is defined in the time domain according to the following steps: (a) write the differential equations governing the cumulants of every order of the response by means of Kroneker algebra [2,6,8]. The latter are a set of first-order linear differential equations, which become a set of algebraic ones for stationary input; (b) evaluate the stochastic sensitivity of the response, once the cumulant differential equations are derived with respect to structural parameters, thus leading to a set of first-order linear differential equations which become algebraic ones for stationary input. Moreover, in this paper, the proposed time domain stochastic sensitivity approach has been applied for evaluating the sensitivity of the response of a primary system with respect to mass, stiffness and damping changes of a secondary system for non-Gaussian stochastic input.

2. Preliminary concepts

Let an n -degree of freedom (n -DOF) dynamical linear system equation of motion be cast in the form

$$M\ddot{\mathbf{U}}(t) + C\dot{\mathbf{U}}(t) + K\mathbf{U}(t) = \mathbf{V}_0\mathbf{W}(t), \tag{1}$$

where M , C and K are the $n \times n$ mass, damping and stiffness matrices, respectively, \mathbf{U} is the n -vector of nodal displacements, \mathbf{V}_0 is a load influence $n \times p$ matrix and $\mathbf{W}(t)$ is a p -vector of forcing functions. Let the vector of state variables $\mathbf{Z}(t)$ be introduced in the form

$$\mathbf{Z}(t) = \begin{pmatrix} \mathbf{U}(t) \\ \dot{\mathbf{U}}(t) \end{pmatrix}. \tag{2}$$

Eq. (1) can then be written in the form

$$\dot{\mathbf{Z}}(t) = \mathbf{D}\mathbf{Z}(t) + \mathbf{V}\mathbf{W}(t), \tag{3}$$

where

$$\mathbf{D} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{V}_0 \end{pmatrix}. \tag{4}$$

Let us suppose now that the k th element $W_k(t)$ of the vector excitation $\mathbf{W}(t)$ is a non-normal delta-correlated process; this means that the value $W_k(t_j)$, for any particular time t_j , is statistically independent of the values of $W_k(t)$ at any other times. This implies that the m th order correlation function of $W_k(t)$ has the following expression [1–3]:

$$k_m[W_k(t_1), W_k(t_2), \dots, W_k(t_m)] = q_{m,k}(t_1)\delta(t_2 - t_1)\delta(t_3 - t_1) \cdots \delta(t_m - t_1) \tag{5}$$

with $\delta(\bullet)$ being Dirac’s delta function and $q_{m,k}(t_1)$ being the m th order intensity coefficient. Moreover, that the components of $\mathbf{W}(t)$ are supposed to be uncorrelated, i.e.

$$k_m[W_k(t_1), W_l(t_2), \dots, W_v(t_m)] = 0 \tag{6}$$

for $k \neq l$ or $k \neq v$ or $v \neq l$. A relevant example of delta-correlated process is represented by the Poisson white noise defined as the following impulsive process:

$$W_k(t) = \sum_{r=1}^{N_k(t)} Q_{k,r} \delta(t - t_{k,r}) \tag{7}$$

where $N_k(t)$ is a counting Poisson process, and the pulses of amplitude $Q_{k,r}$ are identically distributed random variables independent of each other and of the time instants $t_{k,r}$; the latter are distributed according to the Poisson law, with mean arrival rate equal to $\lambda_k(t)$; in the case of Poisson white noise, the intensity coefficient $q_{m,k}(t)$ of order m are simply given by

$$q_{m,k}(t) = \lambda_k(t) E[Q_k^m], \quad m = 1, 2, \dots, \tag{8}$$

where $E[\bullet]$ means stochastic average. If $q_{m,k}(t)$ is independent of t for every m , then the process $W_k(t)$ is strongly stationary. In the particular case in which $\lambda_k(t) E[Q_k^2] = \text{const}$ and $\lambda_k(t) \rightarrow \infty$, then the intensity coefficient $q_{m,k}(t)$, with $m > 2$, approaches zero and the process $W_k(t)$ approaches a normal white noise.

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