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SENSITIVITY ANALYSES OF SENSOR LOCATIONS FOR VIBRATION CONTROL AND DAMAGE DETECTION OF THIN-PLATE SYSTEMS

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This paper addresses the sensitivity problem of sensor locations for vibration control and damage detection of thin-plate systems with parameter variation or noise. For vibration control, the technique for robust determination of sensor locations is presented. Based on the spectral condition number of the Hankel matrix, the optimal sensor locations (OSLs) can be determined, and the effect of noise on the OSLs is investigated using the matrix perturbation theory. For damage detection, the damage locations can be determined using the damage index β derived from the curvature modes. The sensitivity analysis of sensor locations on the detection result for systems with parameter variation is presented. Some experiments are carried out to verify the effectiveness of the proposed method.

1. INTRODUCTION

The problem of sensor locations is crucial for system identification [1–4], active vibration control [5–7] and damage detection [8, 9], which require accurate measurement of the responses of the structure. Many methods, such as the minimal energy principle, mode shape independent principle, degree of observability, Fisher information matrix, etc. have been developed for the determination of sensor locations in many contributions [10–13].

In engineering practice, the measured data are always inaccurate because of the existence of parameter variation or noise. In such cases, a question arises naturally, i.e., are the conventional studies on the determination of sensor locations from these measured data still valid, or under what conditions will the determined locations be insensitive to parameter variation or noise during system identification, vibration control or damage detection?

A survey of the literature shows that the sensitivity analysis of sensor locations for system identification has been investigated intensively [2, 14–16]. Kirkegaard and Brincker determined the OSLs for parametric identification of linear structural systems, and discussed the influence of noise on these OSLs [2]. Kammer studied the effects of noise on sensor placement for on-orbit modal identification of large space structures [14]. Fadale *et al.* suggested that erroneous estimates of the parameters can be ameliorated by placing the sensors at points of maximum sensitivity [15].

However, very few papers have addressed the problems of the sensitivity analysis of sensor locations for vibration control or damage detection. Ma *et al.* investigated the effects of parameter variation on vibration control of beam structures, and established the relationship between robust control and the determination of sensor locations [17]. As for the robust determination of sensor locations for the vibration control of thin-plate systems,

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the effect of noise on the OSLs, and the sensitivity analysis of sensor locations to parameter variation for damage detection, no constructive results have been reported.

The aim of this paper is to study these problems systematically. The structure of this paper is organized as follows. In section 2, the condition for robust determination of sensor location for the vibration control of thin-plate systems with parameter variation is derived, the index for optimal assignment of locations is presented, and the method for analyzing the influence of noise on the OSLs is given. The sensitivity analysis of sensor locations for damage detection and the effect of parameter variation are discussed in section 3. In section 4, the experimental investigations are described and analyzed. Finally, conclusions are drawn.

2. SENSITIVITY ANALYSIS OF SENSOR LOCATIONS FOR VIBRATION CONTROL

2.1. FORMULATION OF THE PROBLEM

The differential equation of a thin plate subjected to control force $F \in \Re^s$ is given by

$$m_0(x, y)\frac{\partial^2 w(x, y, t)}{\partial t^2} + D_0\left(\frac{\partial^4 w(x, y, t)}{\partial x^4} + 2\frac{\partial^4 w(x, y, t)}{\partial x^2\,\partial y^2} + \frac{\partial^4 w(x, y, t)}{\partial y^4}\right) + c_0(x, y)\frac{\partial w(x, y, t)}{\partial t} = F(x, y, t), \quad (0 < x \leqslant a, 0 < y \leqslant b), \tag{1}$$

where $w(x, y, t)$, $m_0(x, y)$, D_0 and c_0 are the transverse deflection, the mass per unit area, the flexural rigidity and the damping coefficient of the thin plate respectively. a and b are the dimensions of the plate. Based on the mode superposition theory, the transverse deflection $w(x, y, t)$ can be expressed as

$$w(x, y, t) = \sum_{i=1}^{m}\sum_{j=1}^{n} \phi_{ij}(x, y)\eta_{ij}(t), \tag{2}$$

where $\eta_{ij}(t)$ and $\phi_{ij}(x, y)$ are the ijth modal co-ordinate and shape function respectively. Assuming that the outputs $\mathbf{y}(t)$ are a set of l displacement signals, with the transformation $\mathbf{Q}(t) = \{\eta(t)^{\mathrm{T}}\ \dot{\eta}(t)^{\mathrm{T}}\}^{\prime}$, the system can be written in the state-space form as

$$\dot{\mathbf{Q}}(t) = \mathbf{A}\mathbf{Q}(t) + \mathbf{B}\mathbf{U}(t), \qquad \mathbf{y}(t) = \mathbf{C}\mathbf{Q}(t), \tag{3}$$

where $\mathbf{U}(t)$ is the vector of control force F,

$$\mathbf{A} = \begin{bmatrix} 0 & & 1 & & \\ & \ddots & & \ddots & \\ & & 0 & & 1 \\ -\omega_{11}^2 & & & -2\zeta_{11}\omega_{11} & \\ & \ddots & & & \ddots \\ & & -\omega_{mn}^2 & & & -2\zeta_{mn}\omega_{mn} \end{bmatrix},$$