A differential equation approach to fuzzy vector optimization problems and sensitivity analysis

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Abstract

The first objective of the analysis presented in this paper is to extend the technique while using differential equations approach for solving fuzzy non-linear programming problem in solving vector optimization problems with fuzzy parameters (VOP-FP). This technique is based mainly on using differential equations approach, which is very effective in finding many local α-Pareto optimal solutions, where the (VOP-FP) is transformed to fuzzy nonlinear autonomous system of differential equations and the relation between the critical points of differential system and local α-Pareto optimal solutions of original optimization problem is proved.

The second objective of the analysis presented in this paper is to obtain sensitivity information for (VOP-FP) by using the technique of trajectory continuation. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

A great deal of work has been done in the field of fuzzy vector optimization problems and sensitivity analysis. From the recent work in this direction, let us mention [2–14]. In [12] Sakawa and Yano presented an interactive decision-making method for multi-objective nonlinear programming problems with fuzzy parameters. In that work, the fuzzy parameters have been characterized by fuzzy numbers and the concept of α-Pareto optimality has been introduced. From the earlier work in this direction by using differential equation approach [10], we presented a technique for solving multi-objective nonlinear programming problems [5], then we extended this result for solving fuzzy nonlinear programming problems [6]. Also, we presented sensitivity analysis for parametric vector optimization problems [7]. In this paper, we shall be concerned with differential equation approach for solving vector optimization problems with fuzzy parameters (VOP-FP) and sensitivity information. This approach is very effective in finding many local α-Pareto optimal solutions.

The paper is organized as follows. In Section 2 we formulate the fuzzy vector optimization problem involving fuzzy parameters in the objective functions. In Section 3, the nonlinear autonomous differential system (Fundamental Equations) for solving (VOP-FP) is introduced. Also, the relation between the critical points

of the differential system which is asymptotically stable and local α-Pareto optimal solutions of the original (VOP-FP) is presented. Finally, in Section 4 a general formula for sensitivity information is also presented. This work is based mainly on the idea of autonomous system of differential equations, by using the technique of trajectory continuation [9,11,14].

2. Problem formulation

In this paper, we consider the following vector optimization problems involving fuzzy parameters in the objective functions in the form:

$$
\begin{aligned}
\text{(VOP-FP):}\quad \min\quad & (f_1(X, \lambda_1), f_2(X, \lambda_2), \ldots, f_m(X, \lambda_m)) \\
\text{s.t.}\quad & X \in M = \{X \in R^n \colon G(X) \leqslant 0\}, \\
& G = (g_1, g_2, \ldots, g_r)
\end{aligned}
\tag{2.1}
$$

where $\Re^n$ is an n-dimensional Euclidean space, $f_i(x, \lambda_i)$, $i = 1, 2, \ldots, m$ and $g_j(X)$, $j = 1, 2, \ldots, r$ possess continuous partial derivatives, where $\lambda_i = (\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{ip_i})$ fuzzy parameters involved in the objective functions. Here the fuzzy parameters are assumed to be characterized by fuzzy numbers as introduced in [2,3,12], for this, a membership function $\mu_{\lambda_{il}}(\lambda)$, $(i = 1, 2, \ldots, m, \ l = 1, 2, \ldots, p_i)$ is defined for a real fuzzy number λ_{ip_i}, where λ_{ip_i} is a convex continuous fuzzy subset of the real line [12]. For simplicity in the notation, we define the following vectors:

$$
\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p), \quad \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p), \quad p \leqslant n.
$$

Definition 2.1. The α-level set of as the number λ_{il}, $(i = 1, 2, \ldots, m, \ l = 1, 2, \ldots, p_i)$ is defined ordinary set $L_\alpha(\lambda)$ for which the degree of their membership functions exceed the level α; $L_\alpha(\lambda) = \{\lambda \colon \mu_{\lambda_{il}}(\lambda_{il}) \geqslant \alpha \ (i = 1, 2, \ldots, m, \ l = 1, 2, \ldots, p_i)\}$. For a certain degree α, problem (1) can be formulated as the following non-fuzzy α-VOP, where

$$
\begin{aligned}
\text{(α-VOP):}\quad \min\quad & (f_1(X, \lambda_1), f_2(X, \lambda_2), \ldots, f_m(X, \lambda_m)) \\
& X \in M, \\
& \lambda \in L_\alpha(\lambda), \\
\text{s.t.}\quad & M = \{X \in \Re^n \colon g_j(X) \leqslant 0, \quad j = 1, 2 \ldots, r\}, \\
& L_\alpha(\lambda) = \{\lambda \colon \mu_{\lambda_{il}}(\lambda) \geqslant \alpha, \quad i = 1, 2, \ldots, m, \ l = 1, 2, \ldots, p_i\}.
\end{aligned}
\tag{2.2}
$$

Definition 2.2. $X^* \in M$ is said to be α-Pareto optimal solution of (α-VOP) iff there does not exist another $(X, \lambda) \in M \times L_\alpha(\lambda)$ such that $f_i(X, \lambda_i) \leqslant f_i(X^*, \lambda_i^*)$, $i = 1, 2, \ldots, m$ with strict inequality holding for at least one i, where the corresponding values of parameter λ^* are called α-level optimal parameters and $\times$ denotes the Cartesian product.

Definition 2.3. $X^* \in M$ is said to be α-Pareto optimal solution of (α-VOP) iff there does not exist another $(X, \lambda) \in M \times L_\alpha(\lambda) \cap N(X^*, \lambda^*; r)$ such that $f_i(X, \lambda_i) \leqslant f_i(X^*, \lambda_i^*)$, $i = 1, 2, \ldots, m$ with strict inequality holding for at least one i, where the corresponding values of parameter λ^* are called α-level optimal locally parameters and $N(X^*, \lambda^*; r)$ denotes the neighbourhood of (X^*, λ^*) with radius r.