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Comput. Methods Appl. Mech. Engrg. 190 (2001) 3865–3879

Computer methods in applied mechanics and engineering

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Recursive formulas for design sensitivity analysis of mechanical systems

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Received 4 December 1998; received in revised form 19 July 1999; accepted 17 May 2000

Abstract

Design sensitivity analysis of a mechanical system is an essential tool for design optimization and trade-off studies. This paper presents a design sensitivity analysis method, using direct differentiation and generalized recursive formulas. The equations of motion are first generated in the Cartesian coordinate system and then transformed into the relative coordinate system by using a velocity transformation. The design-sensitivity equations are derived by directly differentiating the equations of motion. The equations of motion and of design sensitivity are discritized by using the backward difference formula (BDF) in time domain. The resulting equations constitute an overdetermined differential algebraic system (ODAS) and are treated as ordinary differential equations (ODEs) on manifolds. The computational structure of the resulting equations is examined to classify all necessary computations into several categories. The generalized recursive formula for each category is then developed and applied whenever such a category of computation is encountered in the equations of motion and of design sensitivity. Since the velocity transformation yields the equations in a compact form and computational efficiency is achieved by the generalized recursive formulas, the proposed method is not only easy to implement but also efficient. A practical example of a vehicle consisting of many joints, bushings, and tires is given to show the efficiency of the proposed method. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

Erdman [1] has developed a design method for special purpose mechanisms. Kinematic equations that are formulated for a specific mechanism are directly used to develop a design process. Design methods for general mechanical systems have been presented in [2–4].

In designing a structural system, numerical optimization has become already a routine procedure. Design sensitivity and optimization methods have been developed for size, shape, configuration, and topology of structural systems [5,6]. The first- and second-order design sensitivity analyses using Trefftz method have been presented by Kita [7]. In [6], the configuration design method has been successfully applied for kinematically driven systems. In contrast to structural design, there exist few general-purpose codes with design-optimization capabilities for mechanical systems. One of the major difficulties is to establish an efficient and reliable way to analyze the design sensitivity of dynamic responses due to a design change. The objective of this research is to develop an efficient and reliable method for the design sensitivity analysis of general mechanical systems.

There are two kinds of methods in developing the governing equations of design sensitivity; direct differentiation method and adjoint-variable method. In the direct differentiation method, the governing

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PII: S0045-7825(00)00303-0

equations are obtained by differentiating the equations of motion and the constraints [4]. The adjoint-variable method was developed in optimal control [8] and involves the forward numerical integration for the dynamic analysis and the backward numerical integration for the sensitivity analysis. Since the backward numerical integration may incur numerical errors, this research employs the direct differentiation method.

There are several methods in defining the generalized coordinates for dynamic analysis of mechanical systems. Cartesian coordinates have been used in several commercial codes [9,10]. The natural-coordinate method [11] uses points and unit vectors as its generalized coordinates. The orientation of a body is represented by unit vectors. Therefore, the orientation matrix is quadratic in the natural coordinates and its Jacobian is linear. To systematically formulate the equations of motion in relative coordinates, Wittenburg [12] proposed the velocity-transformation method. For computational efficiency, Hooker [13] proposed a recursive formulation for the dynamic analysis of a satellite which has a tree topology. He showed that the computational cost of the formulation increases only linearly with respect to the number of bodies. Featherstone [14] also proposed a recursive formulation to calculate the acceleration of robot arms using screw notation. These ideas have been extended by many researches for multibody rigid and flexible systems in [15–17]. Recently the recursive formulation was generalized in [17] to improve both implementation and efficiency.

The first fully three-dimensional applications of the design sensitivity analysis were demonstrated by Mani [18]. The velocity-transformation method was used to derive the governing equations of design sensitivity. Even though the formulations proposed in the previous studies were for the general mechanical systems, their applications were confined to relatively simple problems due to the complexity of the formulations. The formulation complexity problem was resolved by using a computer algebra.

Constrained mechanical systems are represented by differential equations of motion and algebraic constrained constraint equations, which are often called the ODAS. Several solution methods have been proposed to solve the ODAS in [19–24]. In particular, the parameterization method treated the ODAS as an ordinary differential equations (ODEs) on the kinematic constraint manifolds of the system. The stability and convergence of the method were proved in [24].

This paper employs the velocity-transformation method [12] to derive the governing equations of motion and design sensitivity. Since the virtual displacement and acceleration relationships between the Cartesian and relative coordinates are substituted simultaneously in the velocity transformation method, the governing equations will appear in a compact matrix form. Note that the matrix operations can be computed in a recursive way. Therefore, the matrix form not only makes it easy to debug and understand the computer program but also assures computational efficiency by using the generalized recursive formulas [17].

The recursive kinematic relationships are derived, then generalized in Section 2. The governing equations of design sensitivity and their solution method are presented in Section 3. A set of generalized recursive formulas is derived and applied to evaluate the terms in the equation of motion and design sensitivity in Section 4. A numerical example is presented in Section 5. Finally, conclusions are drawn in Section 6.

2. Relative coordinate kinematics and recursive formulas

2.1. Coordinate systems and relative kinematics

Consider a pair of bodies, as shown in Fig. 1. The X–Y–Z is the inertial reference frame and primed coordinate systems are the body reference frames. Double primed coordinate systems denote the joint reference frames. The orientation of the body reference frame is denoted by $\mathbf{A}$.

Translational and angular velocities of the x'_i–y'_i–z'_i frame, the reference frame for body i, in the X–Y–Z frame are, respectively, defined as $\dot{\mathbf{r}}_i$ and $\boldsymbol{\omega}_i$. Twist velocity in the x'_i–y'_i–z'_i frame is defined as

$$\mathbf{Y}_i \equiv \begin{bmatrix} \dot{\mathbf{r}}'_i \\ \boldsymbol{\omega}'_i \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\mathrm{i}}^{\mathrm{T}} \dot{\mathbf{r}}_i \\ \mathbf{A}_{\mathrm{i}}^{\mathrm{T}} \boldsymbol{\omega}_i \end{bmatrix}. \tag{1}$$