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DESIGN AND SENSITIVITY ANALYSIS OF AN INPUT SHAPING FILTER IN THE *Z*-PLANE

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A variety of input shaper has been proposed to reduce the residual vibration of flexible structures. However, the complexity to solve the non-linear simultaneous equations increases due to addition of the constraints to improve the robustness of an input shaper. In this paper, by proposing a graphical approach which places the shaper zeros on the *z*-plane, the input shaper could conveniently be designed even if the constraints are added for further robustness. Furthermore, it is shown that the proposed method could be accommodated with the variations of both natural frequency and damping ratio. With a mass–damper–spring model, a better performance is obtained using the proposed new multi-hump input shaper.

1. INTRODUCTION

It is necessary to reduce the residual vibration of a light and flexible system for point-to-point operations such as motions of flexible manipulators, ceiling cranes, etc. For vibration reduction, either a feedback or an open-loop control approach could be employed. Instead of using a feedback control, it is possible to eliminate residual vibration with only an open-loop approach. One of the open-loop approaches is the input-shaping technique that employs the convolution of the input shaper with the reference command for vibration reduction.

Smith [1] introduced the posicast control as an input shaper. Recently, Singer and Seering [2, 3] suggested a design method to increase the robustness of Smith's method, using the multiple impulses. Magnitudes and time intervals are obtained by the differentiation of the residual vibration magnitudes with respect to frequency. Singhose, Seering and Singer [4–6] proposed a modified input-shaping technique introducing a multi-hump Extra Insensitive (EI) shaper for robustness improvement. Unfortunately, the calculations are complicated because simultaneous non-linear equations are involved. For damped systems, no analytic solution has been found, so that the solution is numerically presented [7, 8]. Murphy [9] derived an arbitrary rate digital shaping filter from an input-shaping technique. Tuttle and Seering [10] proposed a design method in discrete time domain to cancel out the poles of the systems natural frequencies by the zeros of the input-shaper. They showed that the robustness is improved by adding shaper zeros to the system poles. The shaper provides the smallest time interval with the positive impulse magnitudes. However, their approach should solve the more complicated non-linear simultaneous equations to improve the robustness of the input shaper with the additional constraints. The reason why the difficulty increases is that the sensitivity function of the input shaper is analyzed continuously even if the input shaper is discretely employed.

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In this paper, a method to design the input shaper on the z-plane is proposed by considering the sensitivity function in the discrete time domain. A design procedure of input shapers in both continuous and discrete time domains is reviewed in the next section. The sensitivity equation of the input shaper in the discrete time domain is derived in section 3. In section 4, a brief review of the multi-hump input shaper is given and a method to design a multi-hump input shaper in the discrete time domain is presented. It will be shown that the proposed method has better performance than the existing multi-hump shaper by accounting for the variations of natural frequency and damping ratio simultaneously. In section 5, the performances of multi-hump input shapers in discrete time domain are illustrated.

2. INPUT SHAPER

2.1. CONTINUOUS TIME DOMAIN

At first, the input-shaping technique is briefly reviewed. As an illustration, the input-shaping technique is convolving impulse train with the reference input to eliminate residual vibration. In Figure 1, the responses are shown if a mass–damper–spring system is discretely excited with impulse magnitudes A_0 and A_1. As shown in Figure 1, the first impulse excites the system and then the residual vibration is cancelled out if the second impulse with a damped magnitude A_1 is applied appropriately after half the period of time. In the design of the input shaper, the natural frequency and damping ratio of a system are important factors.

The input shaper is determined from a set of constraint equations that limit the residual vibration of the system. The constraints on vibration amplitude can be expressed as the ratio of residual vibration amplitude with shaping to that without shaping. The ratio [2, 7, 8] is expressed as

$$V(\omega, \zeta) = \mathrm{e}^{-\zeta\omega t_n} \sqrt{\left\{ \sum_{i=0}^{n} A_i \mathrm{e}^{\zeta\omega t_i} \sin\left(\omega t_i \sqrt{1-\zeta^2}\right) \right\}^2 + \left\{ \sum_{i=0}^{n} A_i \mathrm{e}^{\zeta\omega t_i} \cos\left(\omega t_i \sqrt{1-\zeta^2}\right) \right\}^2}, \tag{1}$$

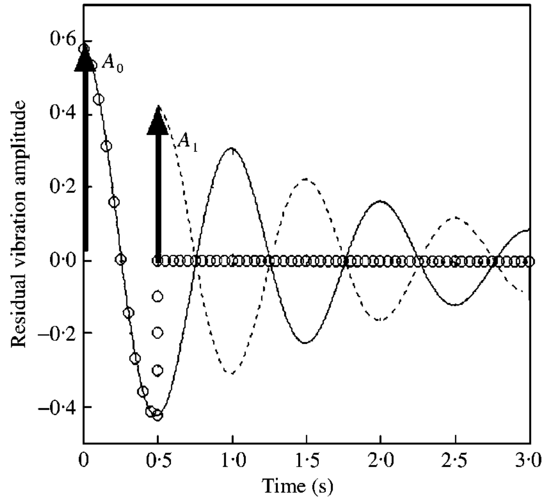

Figure 1. Vibration cancellation using two impulses. ——, Response to A0; - - -, Response to A1; and ○, Response to A0 & A1.