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Sensitivity analysis of coincident critical loads with respect to minor imperfection

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Abstract

A new formulation is presented for sensitivity analysis of a coincident critical load factor. Only symmetric elastic structures subjected to symmetric loads are considered, and sensitivity coefficients are found for a symmetric design modification which corresponds to a minor imperfection. It is shown that the formulation for sensitivity analysis of multiple linear buckling load factor can be successfully combined with that of nonlinear buckling to develop a formulation for coincident nonlinear buckling load factor. The proposed formulation is verified by analytical examples of simple spring–bar systems. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Sensitivity of buckling load factor of an elastic structure with respect to an asymmetric imperfection has been extensively investigated and general forms of imperfection sensitivity analysis have been presented for distributed parameter structures (Koiter, 1945) and for finite dimensional structures (Thompson, 1969; Thompson and Hunt, 1973). It has also been pointed out that imperfection sensitivity can increase due to interaction of buckling modes if two or more critical points coincide or are closely spaced (Ho, 1974; Huseyin, 1975; Thompson and Hunt, 1973; Hutchinson and Amazigo, 1967).

Most of all the studies on sensitivity analysis of buckling load factor, however, concern reduction factor of the critical load due to asymmetric imperfection that is classified as *major imperfection*. Evaluation of sensitivity with respect to a major imperfection is very important to estimate maximum load factor of a structure that has an unavoidable manufacturing or construction error.

In the engineering practice, structures to be built often have symmetry properties, and variation of critical load factor with respect to a symmetric design modification is to be evaluated in the process of redesign or design modification. For a symmetric structure subjected to a set of symmetric loads, which is called *symmetric system* for brevity, symmetric design modification is conceived as *minor imperfection*.

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Roorda (1968) derived analytical formulation of sensitivity analysis of bifurcation load factor with respect to a minor imperfection, and showed that the sensitivity coefficient is bounded for such an imperfection. Ohsaki and Uetani (1996) presented three approaches for sensitivity analysis of bifurcation load factors of symmetric systems, one of which is an explicit form of the formulation by Roorda (1968).

In the field of optimum design, on the other hand, extensive research has been made for computing sensitivity coefficients of linear buckling load factor with respect to design variables such as stiffnesses and nodal locations. Such sensitivity coefficients are called *design sensitivity coefficients* which are used for optimum design and simply for redesigning process. Note that imperfection sensitivity and design sensitivity are virtually identical, and same formulations should be developed. Ohsaki and Nakamura (1994) incorporated the method of imperfection sensitivity analysis for finding optimum designs for specified limit point load factor.

It is well known that optimum designs for specified linear buckling load factor often have multiple or repeated buckling load factors that are nondifferentiable; only directional derivatives or subgradients (Mistakidis and Stavroulakis, 1998) can be defined (Masur, 1984; Haug et al., 1986). Several algorithms have been presented for design sensitivity analysis of multiple buckling load factors (Seyranian et al., 1994), and optimum designs have been obtained for plates and shells under constraints on linear buckling load factor. It has been well discussed in the field of stability analysis that optimization for nonlinear buckling results in coincident buckling that may dramatically increase imperfection sensitivity (Huseyin, 1975; Thompson and Lewis, 1972). Ohsaki (2000) demonstrated that optimization does not always increase imperfection sensitivity even if the optimal design has coincident critical points.

In this paper, a new formulation is presented for sensitivity analysis of coincident buckling load factor of symmetric systems with respect to symmetric design modification which is classified as minor imperfection. The validity of the proposed formulation is demonstrated through the examples of two- and three-degree-of-freedom spring-bar systems.

2. Coincident critical points

Consider an elastic structure discretized by using the finite element method. The vector of proportional loads $\mathbf{P}$ is defined by the vector $\mathbf{P}^0$ of loading pattern and the load factor Λ as

$$\mathbf{P} = \Lambda \mathbf{P}^0. \tag{1}$$

The vector of nodal displacements is denoted by $\mathbf{Q} = \{Q_i\}$. The total potential energy is a function of $\mathbf{Q}$ and Λ which is written as $\Pi^{\mathrm{S}}(\mathbf{Q}, \Lambda)$.

Let S_i denote partial differentiation of Π^{S} with respect to Q_i. Stationary condition of Π^{S} with respect to Q_i leads to the following equilibrium equations:

$$S_i = 0, \quad (i = 1, 2, \ldots, f), \tag{2}$$

where f is the number of degree of freedom of displacements. The path of equilibrium state that originates the undeformed initial state is called fundamental equilibrium path.

The second-order partial differential coefficient of Π^{S} with respect to Q_i and Q_j is denoted by S_{ij}. The matrix $\mathbf{S} = [S_{\mathrm{ij}}]$ is called stability matrix or tangent stiffness matrix. The rth eigenvalue $\lambda^r(\Lambda)$ and eigenvector $\Phi^r(\Lambda)$ of $\mathbf{S}$ along the fundamental equilibrium path are defined by

$$S_{ij}\phi_j^r = \lambda^r \phi_i^r \quad (i = 1, 2, \ldots, f) \tag{3}$$

where ϕ_i^r is the ith component of Φ^r, and the summation convention is used *only* for the subscripts; the superscripts are *not* summed. The eigenmode Φ^r is normalized by