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Design sensitivity analysis and optimization of the hydroforming process

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Abstract

Tube hydroforming has recently been drawing the attention of the automotive industries due to its several advantages over conventional methods. It can produce a wide range of products such as sub-frames, engine cradles, and exhaust manifolds with cheaper production cost by reducing the overall number of processes. The tube hydroforming process is based on the use of internal pressure combined with axial load. Successful tube hydroforming depends on the reasonable combination of the internal pressure and the axial load at the tube ends. This paper deals with the optimal process design (internal pressure and axial load) of the hydroforming process using numerical simulation by the explicit finite element code combined with an optimization tool. An optimization technique is used in order to minimize the tube thickness variation by determining the optimal loading path in the tube expansion forming and the sub-frame forming process. The optimization is performed by means of a gradient-based method including sensitivity analysis. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Hydroforming; Sensitivity analysis; Optimization

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1. Introduction

Tube hydroforming has recently been drawing the attention of the automotive industries due to its advantages over conventional methods. It can produce a wide range of products including sub-frames, engine cradles and exhaust manifolds with cheaper production cost by reducing the overall number of processes. It is mainly used to produce tube-like products with varying cross-sectional shapes along the length direction. Internal hydraulic pressure is applied to expand the tube to fill the cavity of the die, which is designed to yield the shape of the final product. Tube hydroforming starts from a tube that has been precut into the proper length. The tube requires pre-bending as a preforming process. The tube must be bent to the approximate centerline of the finished part prior to hydroforming to enable the tube to be placed in the die cavity. During the bending process, the tube undergoes considerable deformation including thinning. The tube is then placed into the die and the die closed. Hydraulic liquid fills the tube with two side cylinders then closing the ends of the tube. Simultaneously, the liquid is pressurized and the cylinders are pushed in from the side. The material of the tube yields and flows into die cavity and the part is formed.

The tube hydroforming process requires precise control of various forming conditions such as die closing, internal pressure, end sealing and axial feeding. Bursting takes place when pressure is applied too rapidly without enough material feeding, while too much feeding of material tends to cause buckling. Thus successful tube hydroforming depends on the reasonable combination of the internal pressure and axial feeding at the tube ends [1–4].

Many of the problems related to the improvement of the product quality and production efficiency can be directly associated with the optimization procedures. Efficient optimization procedures, integrating the classical mathematical methods of optimization with the finite element method, have recently been developed and applied to structural engineering and to the area of metal-forming [5,6].

Sensitivity analysis has, in the last decade, become a subject of increasing interest with regard to the metal-forming process. Numerical technique of sensitivity analysis plays a crucial role in the optimization procedure of computational engineering problems. Several papers have been published in recent years that are concerned with the parametric sensitivity analysis of elasto-plastic and elasto-viscoplastic structures. The efforts of their authors have been focused on the sensitivity of elasto-plastic structures with respect to variations of material constants, thickness, cross-sectional area and parameters defining the initial geometry [7,8].

This paper deals with sensitivity analysis combined with the explicit finite element code and the optimization of the hydroforming process with respect to process parameters

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such as the internal pressure and axial displacement. For the simulation of the hydroforming process, the shell element, proposed by Belytschko–Tsay [9,10] is used. An anisotropic elastic–plastic material model is employed. To obtain design sensitivity, the direct differentiation method and the updated Lagrangian formulation are used. An optimization technique is used in order to minimize the tube thickness variation by determining the optimal loading path in the tube expansion forming and the sub-frame forming process.

2. Mathematical formulation

The differential equation governing the motion of a material point is:

$$\sigma_{ij,j} + \rho b_i = \rho a_i \tag{1}$$

$$\sigma_{ij} n_j = t_i \quad \text{on}\, S_1 \tag{2}$$

$$u_i = \bar{u}_i \quad \text{on}\, S_2 \tag{3}$$

where σ_{ij} is the Cauchy stress tensor, ρ the current density, a_i the acceleration, b_i the body force density, t_i the given surface traction, $\bar{u}_i$ the displacement boundary condition, and n_j the unit outward normal to the boundary surface.

The variational equation is then given by the following:

$$\int_V \rho a_i \delta u_i \,\mathrm{d}V + \int_V \sigma_{ij} \delta u_{i,j} \,\mathrm{d}V - \int_V \rho b_i \delta u_i \,\mathrm{d}V - \int_{S_1} t_i \delta u_i \,\mathrm{d}S = 0 \tag{4}$$

where δu_i is an arbitrary variation of the displacement field compatible with the boundary condition.

Spatial discretization is performed with n elements, and then the finite element equation is obtained as follows:

$$\sum_{m=1}^{n} \left\{ \int_{V_m} \rho N^t N a \,\mathrm{d}V + \int_{V_m} B^t \sigma \,\mathrm{d}V - \int_{V_m} \rho N^t b \,\mathrm{d}V - \int_{S_1} N^t t \,\mathrm{d}S \right\} = 0 \tag{5}$$

where N is the interpolation matrix, and B the strain–displacement matrix.

By mass lumping of the first term of Eq. (5), the uncoupled finite element equation is obtained as follows:

$$Ma = F^{\text{ext}} - F^{\text{int}} \tag{6}$$

where F^{ext} is the nodal force vector resulting from the surface traction and the body force, F^{int} the nodal force vector resulting from the stress divergence, and M the lumped nodal mass matrix.

The central difference method is used to integrate the equations of motion:

$$a_n = M_n^{-1}(F_n^{\text{ext}} - F_n^{\text{int}}) \tag{7}$$

$$v_{n+1/2} = v_{n-1/2} + a_n \Delta t_n \tag{8}$$

$$u_{n+1} = u_n + v_{n+1/2} \Delta t_{n+1/2} \tag{9}$$

$$\Delta t_{n+1/2} = \tfrac{1}{2}(\Delta t_n + \Delta t_{n+1}) \tag{10}$$

where v and u are the global nodal velocity and displacement vectors, respectively. The geometry is updated by adding the displacement increments to the initial geometry.

3. Sensitivity analysis

The sensitivity coefficients (derivative of the objective function and constraints with respect to the process parameters) are evaluated by the direct differentiation of Eq. (7) [11]:

$$\frac{\partial a_n}{\partial p} = M_n^{-1} \left\{ \frac{\partial F_n^{\text{ext}}}{\partial p} - \frac{\partial F_n^{\text{int}}}{\partial p} - \frac{\partial M_n}{\partial p} a_n \right\} \tag{11}$$

$$\frac{\partial v_{n+1/2}}{\partial p} = \frac{\partial v_{n-1/2}}{\partial p} + \frac{\partial a_n}{\partial p} \Delta t_n \tag{12}$$

$$\frac{\partial u_{n+1}}{\partial p} = \frac{\partial u_n}{\partial p} + \frac{\partial v_{n+1/2}}{\partial p} \Delta t_{n+1/2} \tag{13}$$

where p is a typical internal pressure or axial displacement parameter.

The derivatives of mass matrix, external force vector and internal force vector are expressed as follows [11]:

$$\begin{aligned} \frac{\partial F_n^{\text{ext}}}{\partial p} = {} & \int_{S_1} \frac{\partial N^t}{\partial p} t \,\mathrm{d}S + \int_{S_1} N^t \frac{\partial t}{\partial p} \mathrm{d}S + \int_{S_1} N^t t \frac{\partial(\mathrm{d}S)}{\partial p} \\ & + \int_V \frac{\partial \rho}{\partial p} N^t b \,\mathrm{d}V + \int_V \rho \frac{\partial N^t}{\partial p} b \,\mathrm{d}V \\ & + \int_V \rho N^t \frac{\partial b}{\partial p} \mathrm{d}V + \int_V \rho N^t b \frac{\partial(\mathrm{d}V)}{\partial p} \end{aligned} \tag{14}$$

$$\frac{\partial F_n^{\text{int}}}{\partial p} = \int_V \frac{\partial B^t}{\partial p} \sigma \,\mathrm{d}V + \int_V B^t \frac{\partial \sigma}{\partial p} \mathrm{d}V + \int_V B^t \sigma \frac{\partial(\mathrm{d}V)}{\partial p} \tag{15}$$

$$\begin{aligned} \frac{\partial M_n}{\partial p} = {} & \int_V \frac{\partial \rho}{\partial p} N^t N \,\mathrm{d}V + \int_V \rho \frac{\partial N^t}{\partial p} N \,\mathrm{d}V + \int_V \rho N^t \frac{\partial N}{\partial p} \mathrm{d}V \\ & + \int_V \rho N^t N \frac{\partial(\mathrm{d}V)}{\partial p} \end{aligned} \tag{16}$$

where:

$$\frac{\partial B}{\partial p} = -\mathbf{J}^{-1} \frac{\partial \mathbf{J}}{\partial p} B \tag{17}$$

$$\frac{\partial(\mathrm{d}S)}{\partial p} = \frac{\partial |\mathbf{J}|}{\partial p} \mathrm{d}\xi \,\mathrm{d}\eta \tag{18}$$

$$\frac{\partial(\mathrm{d}V)}{\partial p} = \frac{\partial(h \,\mathrm{d}S)}{\partial p} = \frac{\partial h}{\partial p} \mathrm{d}S + h \frac{\partial(\mathrm{d}S)}{\partial p} \tag{19}$$

$$\frac{\partial |\mathbf{J}|}{\partial p} = |\mathbf{J}| \,\mathrm{tr}\left(\mathbf{J}^{-1} \frac{\partial \mathbf{J}}{\partial p} \right) \tag{20}$$

and x is the position vector of a generic point of the shell, $\mathbf{J}$ the Jacobian of the iso-parametric mapping, $|\mathbf{J}|$ the determinant of Jacobian and h the thickness of the shell.

To obtain the stress sensitivity, consider the anisotropic elastic–plastic material model, of which the constitutive