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Sensitivity analysis and optimization corresponding to a degenerate critical point

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Abstract

Sensitivity coefficients of a critical load factor corresponding to a degenerate critical point is shown to be unbounded even for a minor imperfection excluding very restricted case where the imperfection does not have direct effect on the lowest eigenvalue of the stability matrix. This fact leads to serious difficulty in obtaining optimum design under nonlinear stability constraints. The optimum design problem is alternatively formulated with constraint on the lowest eigenvalue of the stability matrix, and the sensitivity formula for the lowest eigenvalue is presented. The existence of a degenerate critical point and accuracy of the sensitivity coefficient are discussed through the example of a four-bar truss. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Sensitivity analysis of critical load factor of elastic conservative systems has been extensively studied since the pioneering work by Koiter (1945). The purpose of those studies is to present asymptotic formulas for quantitative evaluation of the critical loads with respect to initial imperfection due to, e.g., manufacturing errors and material defects.

There have also been many studies for design sensitivity analysis in the field of optimum design (Haug et al., 1986). Those studies are intended to make analytical and qualitative evaluation of the change in the response due to modification of design variables such as stiffness and nodal location of a skeletal structure. Obviously, imperfection sensitivity and design sensitivity are identical in mathematical sense. Ohsaki and Nakamura (1994) presented an optimum design method based on imperfection sensitivity analysis of limit point loads. For a bifurcation point, it is well known that the sensitivity for a major imperfection is not bounded (Koiter, 1945; Thompson, 1969; Thompson and Hunt, 1973). Therefore, those formulas cannot been used for obtaining optimum designs.

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Structures in practical application, however, usually have symmetry properties, and often reaches an unstable symmetric bifurcation point as the load factor is increased. In this case, antisymmetric imperfection corresponds to a major imperfection that leads to large reduction of the maximum load, and the critical point of an imperfect system is a limit point if the bifurcation points do not coincide. A symmetric imperfection of a symmetric structure, however, is conceived as minor imperfection that does not lead to rapid decrease of the maximum load factor (Roorda, 1968). In this case, the sensitivity coefficients of the bifurcation loads are bounded, and the critical point of the imperfect system remains to be a bifurcation point. In the terminologies of design sensitivity analysis and optimization, a symmetric design modification that is equivalent with a symmetric imperfection is considered as minor imperfection. Ohsaki and Uetani (1996) presented a numerical approach for sensitivity analysis of buckling loads corresponding to a minor imperfection, and applied it for optimizing imperfection sensitive structures (Ohsaki et al., 1998).

The eigenvalues of the tangent stiffness matrix are called stability coefficients in the field of stability analysis. A critical point that has multiple null lowest stability coefficients is called *coincident critical point* (Ho, 1974; Huseyin, 1975; Thompson, 1969). The method by Ohsaki and Uetani (1996) is valid only for a *discrete critical point* where only one stability coefficient vanishes. On the other hand, a *simple critical point* is defined such that the rate of the lowest stability coefficient along the fundamental equilibrium path does not vanish at the discrete critical point (Thompson and Hunt, 1973). The critical point may not be simple even if it is discrete. In this paper, a discrete critical point that is not simple is called *degenerate critical point* which exhibits a *lip singularity* in the terminology of catastrophe theory (Poston and Stewart, 1978; Saunders, 1980).

Existence of a degenerate critical point does not lead to any significant trouble in the sense of instability of the structure, because the lowest stability coefficient immediately increases from zero as the load factor is increased from the critical value. For an optimum design problem, however, a degenerate critical point leads to serious difficulty in the formulation and solution procedure. Ohsaki (2000) presented sensitivity formula with respect to a minor imperfection that can be used even for a coincident critical point including bifurcation and limit points. The formula, however, is not valid for the degenerate critical point.

In this paper, the sensitivity coefficient of a critical load factor corresponding to a degenerate critical point is shown to be unbounded even for a minor imperfection. A new formulation is presented for the optimum design problem under stability constraint. The accuracy of the proposed sensitivity formulation is demonstrated by the example of a four-bar truss.

2. Nonlinear stability analysis

Consider an elastic conservative system subjected to quasi-static proportional loads $\boldsymbol{P} = \Lambda \boldsymbol{P}^0$, where $\boldsymbol{P}^0$ is the base vector and Λ is the load factor. The vector of generalized displacements is denoted by $\boldsymbol{Q} = \{Q_i\}$. The total potential energy is defined in terms of $\boldsymbol{Q}$ and Λ as $\Pi^S(\boldsymbol{Q}, \Lambda)$. Partial differentiation of Π^S with respect to Q_i is written as S_i. Equilibrium condition is derived from the stationary condition of $\Pi^S(\boldsymbol{Q}, \Lambda)$ as

$$S_i = 0 \quad (i = 1, 2, \dots, f), \tag{1}$$

where f is the number of degree of freedom of the system.

In the following, the summation convention is used only for the subscript. Second partial differentiation of Π^S with respect to the displacements is written as S_{ij}. The rth eigenvalue or stability coefficient $\lambda^r(\Lambda)$ and eigenvector $\Phi^r(\Lambda)$ of the stability matrix $\boldsymbol{S} = [S_{ij}]$ are defined by

$$S_{ij}\phi_j^r = \lambda^r \phi_i^r \quad (i = 1, 2, \dots, f), \tag{2}$$

where ϕ_i^r is the rth component of Φ^r that is normalized by