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Mathematical Social Sciences 42 (2001) 179–201

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# The life-cycle model of saving with uncertain lifetime and borrowing constraint; characterization and sensitivity analysis

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Received 1 April 2000; received in revised form 1 August 2000; accepted 1 December 2000

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## Abstract

This paper investigates the characterization and sensitivity analysis of the life-cycle model of saving with uncertain lifetime and borrowing constraint first formulated by Yaari [Review of Economic Studies 32 (1965) 137–150]. Under mild conditions, terminal wealth depletion is shown to be an intrinsic property of the model. A three-step procedure for the sensitivity analysis is presented. While the presence of an endogenous wealth depletion time complicates the sensitivity analysis, incorrect and misleading sensitivity results will be obtained if the existence of terminal wealth depletion is ignored. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* Life-cycle model; Saving; Uncertain lifetime; Wealth depletion; Sensitivity analysis

*JEL classification:* C61; D11; D91; E21

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## 1. Introduction

The life-cycle model of saving with uncertain lifetime and borrowing constraint, first formulated by Yaari (1965), has been extensively studied and utilized in many different areas in economics. In spite of its popularity, some fundamental properties of the model have not yet been fully discovered and explored. In a recent study, Leung (1994) derives a new characterization of the model. Without bequest motives and life annuities, the optimal solution to the life-cycle optimization problem is shown to entail a zero-wealth period that lasts through the maximum lifetime. The new characterization is not only of

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theoretical interest because several simulation studies have established that this type of wealth depletion can occur very early (Mirer, 1994; Leung 1994, 2000a). In addition, it provides a rigorous explanation for the empirical finding of early wealth depletion (Hurd, 1989) as well as a solution to the puzzle of widespread low wealth holdings (Leung, 2000a).

In light of the theoretical significance and empirical relevance of terminal wealth depletion, a more comprehensive analysis of the causes and consequences of wealth depletion is warranted. In this paper, I further characterize the life-cycle model by examining the determinants of wealth depletion and investigating how sensitivity analysis can be conducted in the presence of wealth depletion. Firstly, I generalize the results in Leung (1994) by offering weaker sufficient conditions for the existence of the wealth depletion time. Secondly, I examine explicitly how the wealth depletion time is determined and provide some conditions to guarantee the uniqueness of the wealth depletion time. Thirdly, I provide a three-step procedure to derive the comparative statics of the wealth depletion time as well as the comparative dynamics of consumption and wealth. The comparative statics analysis reveals how the wealth depletion time varies with the parameters of the model. Since the paths of consumption and wealth depend on an endogenous wealth depletion time, the comparative dynamics are slightly more complicated than those of the traditional life-cycle models because variations in the parameters of the model will have a *direct* effect on the paths of consumption and wealth as well as an *indirect* effect through the wealth depletion time. Although the mathematical derivations are fairly involved, I illustrate how sensitivity results can be obtained in a tractable way.

The paper has two special objectives. The first objective is to derive tractable comparative statics and dynamics results. In most of the studies employing the life-cycle model of saving with uncertain lifetime and borrowing constraint (e.g. Hubbard and Judd (1987)), it is customary to study the properties of the model by means of numerical simulations because tractable analytical results are hard to come by. While simulation analysis is valuable, it may even be better if analytical results are available to guide the analysis and interpret the findings. Although the mathematical derivations appear complicated, I show that tractable comparative statics and dynamics can be obtained. The second objective is to study the role of terminal wealth depletion in the sensitivity analysis. How important is the wealth depletion result in characterizing the paths of consumption and wealth? Previous studies on the life-cycle model of saving with uncertain lifetime and borrowing constraint (e.g. Hurd (1989)) have failed to consider the existence of terminal wealth depletion, thus the implications of the new characterization have yet to be discovered. I will investigate whether incorrect sensitivity results will be obtained if the wealth depletion time is ignored.

The plan of the paper is as follows. Section 2 describes the model and offers sufficient conditions for the existence of the wealth depletion time. Several examples are employed to illustrate the usefulness of the conditions. All the examples are constructed in such a way that the model has an explicit closed-form solution. Section 3 deals with the determination of the wealth depletion time. Section 4 presents the sensitivity analysis. Section 5 concludes the paper. Detailed mathematical proofs are provided in Appendix A.

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