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Sensitivity analysis in fuzzy multiobjective linear fractional programming problem

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Abstract

In this paper, we study measurement of sensitivity for changes of violations in the aspiration level for the fuzzy multiobjective linear fractional programming problem. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In most practical applications of Mathematical Programming the possible values of the parameters required in the modeling of the problem are provided by either a decision maker subjectively or a statistical inference from the past data. In order to reflect this uncertainty, the model of the problem is often constructed with fuzzy data. The approach given by Bellman and Zadeh [1] forms the basis of an overwhelming majority of fuzzy decision-making models in Mathematical Programming.

The first attempt to fuzzify a linear program is due to Zimmermann [17]. In the current literature there are several models of fuzzy linear programming problem (FLP). To solve different models of FLP, different approaches appear in literature such as those given by Delgado et al. [4,5]. For solving fuzzy multiobjective linear program (FMOLP) one may cite the approaches taken by Zimmermann [18] and Llena [10].

The use of parametric programming in FLP problem with fuzzy goal and fuzzy constraint was first proposed by Chanas [2]. Shafai and Sotirov [15] gave some conditions which are both necessary and sufficient for a solution to a FLP problem to be unique in the presence of parameter perturbations.

Sensitivity analysis in FLP problem with crisp parameters and soft constraints was considered first by Hamacher et al. [8] and later on by many others, e.g. Tanaka et al. [16], and Fulle'r [7].

Sensitivity analysis for fuzzy linear fractional programming problem (FLFP) was studied by Dutta et al. [6]. Our objective in this work is to study sensitivity analysis for multiobjective linear fractional programming problem (MOLFP) in fuzzy

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environment. MOLFP models have been the subject of wide interest in recent times owing to their widespread use in the evaluation of a number of important economic activities. Sakawa and Yano [13] have presented a fuzzy approach for solving MOLFP. Sakawa and Yumine [14] gave a new interactive fuzzy satisfying method by the combined use of bisection method and linear programming method to derive the satisfying solution for the decision maker. Luhandjula [11] has given a linguistic approach to MOLFP by introducing linguistic variables to represent linguistic aspirations of the decision maker.

In this paper, a fuzzy multiobjective linear fractional programming problem (FMOLFP) is studied. The problem has been formulated in a multiple objective environment where the objective functions are fuzzy. Accomplishment of objective functions is subject to both fuzzy as well as deterministic (soft) constraints. A sensitivity analysis is carried out for the problem and investigations have been performed on the measurement of sensitivity for change in violations of the aspiration levels for different cases.

2. Problem formulation

The vector-maximum MOLFP is defined as follows:

(*MOLFP*):

$$\underset{x \in X}{\text{maximize}} \quad F(x) = [f_1(x), f_2(x), \ldots, f_k(x)],$$

where $f_i(x) = l_i(x)/m_i(x)$.

Also, $l_i(x) = c_i^{\mathrm{T}} x + \alpha_i$, $m_i(x) = d_i^{\mathrm{T}} x + \beta_i$ are real-valued and continuous functions on X and $m_i(x) \neq 0$ $(i = 1, \ldots, k)$ for all $x \in X$ and $X = \{x \mid Ax = b,\ x \geqslant 0,\ x \in R^n,\ b \in R^m,\ (A)_{m \times n}\}$ is assumed to be nonempty, convex and compact set in R^n.

Definition 1. $\bar{x} \in R^n$ is an efficient solution of MOLFP if there is no $\hat{x} \in R^n$ such that

$$f_i(\hat{x}) \geqslant f_i(\bar{x}), \quad i = 1, 2, \ldots, k$$

and

$$f_i(\hat{x}) > f_i(\bar{x}) \quad \text{for at least one } i.$$

The fuzzy model of MOLFP is stated here under

(*FMOLFP*):

$$\begin{array}{ll} \text{maximize} & F(x)_{\sim} = [f_1(x), f_2(x), \ldots, f_k(x)] \\ \text{subject to} & x \in X, \end{array}$$

where

$$X = [a_i x \leqslant_{\sim} b_i,\ i = 1, 2, \ldots, m_1,\ a_j x \leqslant b_j,\ j = m_1 + 1, \ldots, m].$$

Note: The total number of constraints in the feasible region X is m out of which m_1 is the number of fuzzy constraints and the remaining constraints are deterministic (soft).

Now a solution $x \geqslant 0$ is to be determined in such a way that the fuzzy constraints, $a_i x \leqslant_{\sim} b_i$, $i = 1, \ldots, m_1$, are satisfied as far as possible while the soft constraints $a_j x \leqslant b_j$, $j = m_1 + 1, \ldots, m$, are fully satisfied and also the solution is acceptable with respect to all the k objective functions. It is assumed that either the decision maker should specify the aspiration levels for the objective functions, or we have to define properties of the solution space for 'calibration' of the objective functions. Two cases are considered for the problem FMOLFP.

Case (I): denominators are identical.

Case (II): denominators are different.

Case (I): Using the variable transformation $y = hx$ (h is scalar) Charnes and Cooper [3] replaced a single objective linear fractional programming problem (LFP) by two linear programming problems. Further, Zionts [19] established that if the LFP has a finite solution, then the denominator cannot have two different signs in the feasible region. Therefore, it is sufficient to solve only one of the two linear programming problems in order to get the solution of LFP. Since the variable transformation is one to one, we obtain the equivalent FMOLP problems as follows.

If $m(x) > 0$ the equivalent FMOLP is

$$\begin{array}{ll} \text{maximize}_{\sim} & \{c_1^{\mathrm{T}} y + \alpha_1 h,\ c_2^{\mathrm{T}} y + \alpha_2 h, \ldots, c_k^{\mathrm{T}} y + \alpha_k h\} \\ \text{subject to} & x \in U, \end{array}$$

where

$$\begin{aligned} U = \{(y, h) \mid\ & a_i y - b_i h \leqslant_{\sim} 0,\ i = 1, 2, \ldots, m_1, \\ & a_j y - b_j h \leqslant 0,\ j = m_1 + 1, \ldots, m, \\ & d^{\mathrm{T}} y + \beta h =_{\sim} 1,\ y \in R^n,\ h \in R\}. \end{aligned}$$