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Shape design sensitivity analysis in elasticity using the boundary element method

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Abstract

This paper deals with sensitivity analysis of the different functionals appearing in optimum shape design in elasticity using boundary element method (BEM). First, a general review concerning sensitivity analysis of the most usual functionals in elasticity is presented, based on the continuum approach. The accuracy in sensitivity analysis depends on the accuracy in evaluating strains and stresses on the boundary. A general procedure for strain calculation based upon some results of differential geometry of surfaces is shown. Another essential aspect in sensitivity analysis is the definition of the design velocity on the boundary, which defines the change in the geometry of the elastic solid. A computational treatment independent of the design variables used is presented, defining nodal values of the design velocity and taking advantage of the boundary element approximation. Finally, the feasibility and accuracy of the proposed procedures are assessed through several example problems. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Both the finite element method (FEM) [3,4,10] and the boundary element method (BEM) [1,9,12,16] have been extensively used to solve the optimum shape design problem in elasticity. However, most researchers tend to use BEM because of its advantages in such a problem, both for the discretization and remeshing along the optimization process and its better accuracy in the evaluation of strains and stresses on the boundary, which is an essential aspect in sensitivity calculations [10,2].

Traditionally, two approaches have been used in sensitivity analysis of the different functionals in elasticity. In the discrete approach, an implicit derivation of the discretized set of equations is performed and then a new problem is solved with the same matrix but with a different load vector [9]. Obviously, obtaining the derivatives of the matrix and load vector with respect to each design variable could be difficult depending on the specific problem. An alternative option is the continuum approach, which uses the material derivative of continuum mechanics and the concept of adjoint problem in order to get explicit expressions in terms of the design velocity for the sensitivities of the different functionals [11].

The continuum approach, together with BEM, has been used in recent research to solve the shape design sensitivity analysis problem. Meric [14] analyzed the problem of shape optimization in 2D heat transfer problems, using specific design variables with a semi-analytical treatment of design velocity on the boundary. Tai and Fenner [16] used geometrical design variables with a specific definition of design velocity in shape optimization in 2D elasticity. Erman and Fenner [8] described the shape optimization problem in 3D elasticity based upon an implicit derivation of the BEM integral equations. Burczynski et al. [1] considered the shape optimization problem in 3D elasticity, using different functionals to define both objective and constraint functions, and defining specific adjoint problems. Finally, Kocandrle and Koska [13] studied the shape optimization problem in 3D elasticity, obtaining an explicit expression to evaluate the sensitivity of stress functionals, considering the design variables as fictitious loads applied on the boundary.

However, the different applications make use of either specific functionals or 'ad hoc' design variables for each problem. This paper presents a revision of the sensitivity analysis for the different functionals appearing in shape optimization in elasticity based on the continuum approach, and after that the paper describes a general method to

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calculate strains and stresses on the boundary and a treatment of the design velocity on the boundary in terms of nodal values, taking advantage of the BEM discretization, both for 2D and 3D problems.

2. Sensitivity analysis in elasticity: the continuum approach

Geometry is the true unknown in optimum shape design, so the design process will be an iterative one, with successive changes in geometry in order to obtain the optimum. It is an evolution problem where the new geometry is obtained from the previous geometry in each step and the change depends on the sensitivity concerning objective and constraint functionals with respect to the design variables.

The design objective is usually measured by means of a domain functional such as

$$\psi_\Omega = \int_\Omega f(u_i, \sigma_{ij})\, \mathrm{d}\Omega \tag{1}$$

where f is a function depending on the displacements u_i and stresses σ_{ij} corresponding to the elastic problem. If the material derivative concept is applied to the previous functional the following equation is obtained [10]

$$\dot{\psi}_\Omega = \int_\Omega (f, u_i, \dot{u}_i + f_{,\sigma_{ij}}, \dot{\sigma}_{ij})\, \mathrm{d}\Omega - \int_\Omega \times [(u_{i,j}v_j)f_{,ui} + (\sigma_{ij,k}v_k)f_{,\sigma ij}]\, \mathrm{d}\Omega + \int_\Gamma f v_\mathrm{n}\, \mathrm{d}\Gamma \tag{2}$$

where $f_{,u_i}, f_{,\sigma_{ij}}$ are the partial derivatives of f with respect to u_i and σ_{ij}, respectively; v_j, v_k are the components of the design velocity field (change in the shape of the domain), v_n is the normal component of the velocity field on the boundary, and $\dot{u}_i$, $\dot{\sigma}_{ij}$ are the material derivatives of displacements and stresses, respectively.

If the solution of the elastic problem and the design velocity field are known, it is possible to calculate the last two integrals. To calculate the first integral in Eq. (2), the material derivatives $\dot{u}_i$ and $\dot{\sigma}_{ij}$ (depending on the velocity field) need to be considered. To avoid this inconvenience, an adjoint problem is introduced in terms of the adjoint variable [10], and then the following explicit expression for the material derivative of the functional can be obtained

$$\begin{aligned}\dot{\psi}_\Omega = & -\int_\Omega (\lambda_{i,j}v_j)b_i\, \mathrm{d}\Omega + \int_\Gamma \lambda_i b_i v_\mathrm{n}\, \mathrm{d}\Gamma - \int_\Gamma (\lambda_{i,j}v_j)t_i\, \mathrm{d}\Gamma \\ & + \int_\Gamma [(\lambda_i t_i)_{,j} n_j + H\lambda_i t_i] v_\mathrm{n}\, \mathrm{d}\Gamma + \int_\Omega \sigma_{ij}^\lambda (\epsilon_{ij,k} v_k)\, \mathrm{d}\Omega \\ & + \int_\Omega \sigma_{ij}(\epsilon_{ij,k}^\lambda v_k)\, \mathrm{d}\Omega - \int_\Gamma \sigma_{ij}\epsilon_{ij}^\lambda v_\mathrm{n}\, \mathrm{d}\Gamma \\ & - \int_\Omega [(u_{i,j}v_j)f_{,u_i} + (\sigma_{ij,k}v_k)f_{,\sigma_{ij}}]\, \mathrm{d}\Omega + \int_\Gamma f v_\mathrm{n}\, \mathrm{d}\Gamma \end{aligned} \tag{3}$$

where t_i represents the tractions on the boundary, b_i the external forces per unit volume acting on the solid, σ_{ij} the stresses corresponding to the real problem; λ the adjoint variable (displacement field of the adjoint problem), ϵ_{ij}^λ the strains corresponding to the adjoint problem; and finally H is the curvature of Γ in 2D problems and twice the mean curvature of Γ in 3D problems.

If the design objective is measured by means of a boundary functional such as

$$\psi_\Gamma = \int_\Gamma f(u_i, \sigma_{ij})\, \mathrm{d}\Gamma \tag{4}$$

in a similar way it is possible to obtain

$$\dot{\psi}_\Gamma = \int_\Gamma (f_{,u_i}\dot{u}_i + f_{,\sigma_{ij}}\dot{\sigma}_{ij})\, \mathrm{d}\Omega + \int_\Gamma (Hv_\mathrm{n} + v_{s,s})f\, \mathrm{d}\Gamma \tag{5}$$

where $v_{s,s}$ is the tangential derivative of tangential component of the velocity field on the boundary.

Introducing the corresponding adjoint problem, the following explicit expression could be obtained for the material derivative of the functional

$$\begin{aligned}\dot{\psi}_\Gamma = & -\int_\Omega (\lambda_{i,j}v_j)b_i\, \mathrm{d}\Omega + \int_\Gamma \lambda_i b_i v_\mathrm{n}\, \mathrm{d}\Gamma - \int_\Gamma (\lambda_{i,j}v_j)t_i\, \mathrm{d}\Gamma \\ & + \int_\Gamma [(\lambda_i t_i)_{,j} n_j + H\lambda_i t_i] v_\mathrm{n}\, \mathrm{d}\Gamma + \int_\Omega (\epsilon_{ij,k} v_k)\sigma_{ij}^\lambda\, \mathrm{d}\Omega \\ & + \int_\Omega (\epsilon_{ij,k}^\lambda v_k)\sigma_{ij}\, \mathrm{d}\Omega - \int_\Gamma \sigma_{ij}\epsilon_{ij}^\lambda v_\mathrm{n}\, \mathrm{d}\Gamma \\ & - \int_\Gamma (Hv_\mathrm{n} + v_{s,s})f\, \mathrm{d}\Gamma \end{aligned} \tag{6}$$

In the previous equations, the domain integrals can be transformed into boundary integrals by using the appropriate variational identities [10].

Note that the evaluation of the design sensitivity of functionals (1) and (4) requires the solution of real and adjoint problems. The last one will depend on the function f appearing in the kernel of the integrals, although they are similar problems with different loads [2]. This leads to an efficient calculation by using finite or boundary elements because the same set of equations with different loads has to be solved.

The previous general expressions can be used to obtain the material derivative of the most common functionals appearing in the design of structural components. So, the simplest functional to be considered deals with the amount of material, associated to the volume

$$\dot{\psi}_\Omega = \int_\Omega \mathrm{d}\Omega \tag{7}$$

In this case, $f = 1$, and applying directly the result from Eq. (3)

$$\dot{\psi}_\Omega = \int_\Gamma v_\mathrm{n}\, \mathrm{d}\Gamma \tag{8}$$

and, it is not necessary to define the adjoint problem. The