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Sensitivity analysis and optimization of truss/beam components of arbitrary cross-section II. Shear stresses

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Abstract

This paper presents a general approach for detailed analysis and design optimization of arbitrary cross-sections of truss/beam built up structures. The approach allows arbitrary shape parametrization of 2-D cross-sections, as long as the coordinates of the contour vertices and their velocities are available, and is well suited for integration with existing CAD modelers and FEM analyzers. It leads to an inexpensive 2-D size/shape optimization in an alternative to costly 3-D shape optimizations, virtually impossible for real-life built up structures. Any composite multi-contour cross-section is first discretized with elementary triangles. Direct integration on the surface, using closed-form formulas, allows computation of the cross-section axial properties. Numerical integration on the boundary, along the line segments used to describe the contour, allows the computation of the shear properties. The power-series method is used to obtain the equilibrium equations and their governing linear warping system. The design sensitivities are calculated by the direct differentiation method requiring only backward substitutions on the triangular stiffness matrix. Numerical tests extensively verify the accuracy and the practical use of the formulation and implementation.

Keywords: Cross-section optimization; Design sensitivity; Structural optimization; Shear stress; Power-series

1. Introduction

Although truss/beam like components are extensively used in civil and mechanical engineering structural applications, only very few authors have addressed the topic of design optimization of arbitrary cross-sections. Apostol and Santos [1,2] described an arbitrary parametric cross-section through geometrical shape variables, and expressed each vertex coordinate as a linear combination of these shape variables.

The cross-section axial properties were computed and differentiated analytically with respect to the selected shape variables. These properties, representing finite element input quantities, were used as performance measures in the optimization model.

However, previous research on design optimization of arbitrary cross-sections, has been focused on axial properties and stresses. Still, it is well known that the shear stresses induced by transverse shear loads and especially by torsion can reach significant values for space frames and therefore they can neither be neglected, nor poorly approximated.

The analysis of the torsion problem may be performed by the *semi-inverse* method proposed by Saint-Venant by the mid of the last century. The method is exact, provided that a *warping function* satisfying a differential equation and certain boundary conditions is known. A comprehensive description of the semi-inverse method can be found in Timoshenko [3].

One of the first papers addressing cross-section optimization of elastic bars under torsion seems to belong

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to Banichuk [4], although shear stresses were not considered. Pilkey and Liu [5] avoided finite or boundary elements for the computation of the warping function by using the direct integration of the boundary integral equation instead. Their approach to the solution of the torsion problem was used by Schramm and Pilkey [6] for structural optimization in conjunction with shape description via B-splines. The optimization contained the torsion moment of inertia and the cross-section area only, whereas the transverse shear, the shear center and the shear stresses were not taken into consideration. The same authors extended the previous research to thin-walled beam theory, Schramm et al. [7], describing the shape with non-uniform rational B-splines (NURBS), Schramm and Pilkey [8,9].

A variational approach based on power-series, was introduced by Mindlin [10] for the computation of the warping function. This approach was further improved by Kosmatka [11,12], and it was selected in this paper as the basis for the cross-section analysis and shape optimization. Kosmatka used exact Gauss-quadrature integration for higher order polynomials, for numerical evaluation of the integrals on a surface, according to Dunavant [13].

In this paper, the power-series approach used by Kosmatka is used for numerical evaluation of the warping function. However, the Green's formula is used to convert the surface integrals into contour integrals, see Press et al. [14].

The direct differentiation method is used to differentiate the equilibrium equations with respect to cross-section shape design parameters. The design sensitivities are finally used to demonstrate the validity of the approach for numerical shape optimization of truss/beam arbitrary cross-sections. Although two design parametrization approaches are developed and compared in this paper, the current method allows the use of any general parametrization like those provided by parametric and associative CAD systems.

2. Shape parametrization

The aim of design parametrization is to relate the higher level shape variables with the lower level variables, vertex and control points, used to define the cross-section contours. Once the design parametrization is known, the design sensitivities of the vertex coordinates with respect to the shape variables may be easily computed. This sensitivity information is referred as the cross-section velocity field.

Apostol and Santos [2] used a direct linear parametrization, assuming each vertex coordinate as being a linear combination of shape variables. The approach led to an important simplification, since the constant coefficients of the linear combination represent in fact the required velocity field.

Direct non-linear parametrizations, allowing non-linear dependencies between the shape variables and the vertex coordinates is also sometimes used in practice. Recently, Langelaan and Livne [15] parametrized the distance along the axis and the radii in transversal directions of slender curved hollow tubes, providing analytically the cross-section velocity field. A numerical example is used in this paper to show the disadvantages of this method, mainly related to the non-smoothness of the final shape.

Indirect parametrization, using interpolation curves for the contour definition, assures the smoothness of the cross-section shape. Braibant and Fleury [16] introduced the use of B-splines for shape structural optimization, whereas Schramm and Pilkey [8,9] used NURBS for shape parametrization. However, interpolation curves may also make it difficult to achieve desired regular shapes.

If proper control dimensions (radii, angles, etc.) are selected as shape variables, the final shape remains regular and smooth as desired. Such direct parametrization strategy in connection with advanced CAD systems seems to be attractive for structural optimization, see Lindby and Santos [17].

3. Axial properties

To calculate the axial properties and their design sensitivities with respect to a set of shape parameters **b**, Ref. [2] describes a complex cross-section as the sum of closed polygonal contours. In the initial reference system, the X-axis is along the beam, and the beam transversal plane ZOY is used for locating all vertices used to define the closed contour making-up the cross-section.

The cross-section centroid (z_G, y_G) and the axial moments of inertia related to the reference system ZOY allow the computation of the principal axes of inertia denoted by $\widetilde{X}_2 G \widetilde{X}_3$, with the $\widetilde{X}_1$-axis remaining along the beam. [1] Consequently, I_2 and I_3 become the maximum and the minimum axial moments of inertia.

Fig. 1 shows the dependency between the two reference systems. The principal angle of inertia α_p is conventionally defined as the angle made by the arbitrary Z-direction with the principal maximum axis of inertia $\widetilde{X}_2$. All vertex coordinates are translated and rotated to the principal axes of inertia.

[1] Notation $\sim$ denotes a measure related to the principal system of inertia.