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Design sensitivity analysis of structures based upon the singular value decomposition

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Abstract

The singular value decomposition (SVD) is employed for design sensitivity analysis of structures. As the squares of singular values are the bounds of power, energy and power spectral density ratios between the input and output vectors, shaping the singular values of a structure is equivalent to shaping the response of the structure. Comparison is made of the proposed sensitivity analysis based upon the SVD with the conventional techniques. The issues such as structural robustness, worst loading case and multiple load cases are studied. As shown, design sensitivity analysis based upon the SVD can give good insight into static and dynamic response characteristics of structures; it is more informative than eigenvalue design sensitivity analysis and, in particular, computationally advantageous in case of multiple load cases. Numerical examples are presented to illustrate the proposed approach. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Design sensitivity analysis; Singular values; Singular value decomposition; Worst case loading; Structural robustness; Sensitivity bounds

1. Introduction

One of the most important tools in designing a structure is the sensitivity of a structural system to variations of its parameters. The design sensitivity analysis is concerned with the relationships between the parameters of a system and the system behavior characterized by some performance measures. For the review and exposition of the subject matter, the works [1–3,7–9,11] and [18] can be cited. In design sensitivity studies, the sensitivity of structural response to variations in design variables is investigated by use of some performance measures under some constraints, which is accomplished by solving some algebraic equations, eigenvalue problems or ordinary differential equations. Investigations for the sensitivity analysis of structural response to changes in design parameters typically employ displacement, eigenvalue, eigenvector or stress as performance measures and constraints and, up to date, no attention is paid to the behavior of the singular values and singular vectors of structures. On the other hand, the singular value decomposition (SVD) based analysis is well suited to study input–output directional relationships of systems. In particular, singular values of a structure has a special meaning since the squares of singular values

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are the bounds of power, energy and power spectral density ratios between the input and output vectors; thus, shaping the singular values of a structure is equivalent to shaping the response of the structure. Furthermore, singular vectors tell us how the outputs are related to the inputs.

The SVD is a very powerful tool that is utilized for studying input–output properties in multi-variable control systems, e.g., [4] and [17]. In this paper, under the interaction of computational mechanics and system theory disciplines, the SVD is applied to the design sensitivity analysis of structures. By considering finite element equations, static and dynamic response design sensitivity analyses of structures are investigated by use of the SVD. Comparison is made of the proposed sensitivity analysis based upon the SVD with the conventional techniques. As shown, the SVD based design sensitivity analysis can give good insight into static and dynamic response of structures; it is more informative than eigenvalue design sensitivity analysis and, in particular, computationally advantageous in case of multiple load cases.

The outline of the article is as follows: some properties of the SVD are revisited in Section 2, and then the SVD is introduced to finite element equations of structural dynamics in Section 3. Singular value and singular vector sensitivity relationships are presented in Section 4. The proposed design sensitivity analysis based upon the SVD is implemented into model problems and compared with conventional techniques in Section 5, and conclusions are drawn in Section 6.

2. Properties of the singular value decomposition

Note that the exposition of the material on the SVD is based on that of [6,10,14] and [16]. Consider the matrix $\mathbf{A} \in \mathscr{C}^{m\times n}$, then there exist unitary matrices $\mathbf{U} \in \mathscr{C}^{m\times m}$, $\mathbf{\Sigma} \in \mathscr{R}^{m\times n}$ and $\mathbf{V} \in \mathscr{C}^{n\times n}$ called the SVD of $\mathbf{A}$ such that $\mathbf{A}$ can be factored as

$$\mathbf{A} = \mathbf{U\Sigma V}^{\mathrm{H}}, \tag{1}$$

where the columns of $\mathbf{U} = [\mathbf{u}_1|\mathbf{u}_2|\cdots|\mathbf{u}_m]$ and $\mathbf{V} = [\mathbf{v}_1|\mathbf{v}_2|\cdots|\mathbf{v}_n]$ are, respectively, the left and right singular vectors, and $\mathbf{V}^{\mathrm{H}}$ is the conjugate transpose of $\mathbf{V}$. If $m = n$, then $\mathbf{\Sigma} = \mathrm{Diag}\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$; on the other hand, if $m > n$, then

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{\mathrm{d}} \\ \mathscr{O}_{(m-n)\times n} \end{bmatrix}. \tag{2}$$

If $m < n$, then

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{\mathrm{d}} & \mathscr{O}_{m\times(n-m)}, \end{bmatrix} \tag{3}$$

where $\mathbf{\Sigma}_{\mathrm{d}} = \mathrm{Diag}\{\sigma_1, \sigma_2, \ldots, \sigma_p\}$, $p = \min(m, n)$, $\mathscr{O}_{i\times j} \in \mathscr{R}^{i\times j}$ whose elements are all zero, and σ_i are the singular values of $\mathbf{A}$. Note that $\mathbf{u}_i$ and $\mathbf{v}_i$ are, respectively, orthonormal eigenvectors of $\mathbf{AA}^{\mathrm{H}}$ and $\mathbf{A}^{\mathrm{H}}\mathbf{A}$; namely,

$$\mathbf{UU}^{\mathrm{H}} = \mathbf{I} \text{ and } \mathbf{AA}^{\mathrm{H}}\mathbf{U} = \mathbf{U\Sigma}^2, \tag{4}$$

$$\mathbf{VV}^{\mathrm{H}} = \mathbf{I} \text{ and } \mathbf{A}^{\mathrm{H}}\mathbf{AV} = \mathbf{V\Sigma}^2, \tag{5}$$

where $\mathbf{I}$ is the identity matrix. In addition, for a square matrix $\mathbf{A}$

$$\text{if } \mathbf{A} = \mathbf{U\Sigma V}^{\mathrm{H}}, \quad \mathbf{A}^{-1} = \mathbf{V\Sigma}^{-1}\mathbf{U}^{\mathrm{H}}. \tag{6}$$

Although the singular values of $\mathbf{A}$ are uniquely defined, the singular vectors are not. If $\mathbf{A} = \mathbf{U\Sigma V}^{\mathrm{H}}$, then $\mathbf{A} = \mathbf{U}'\mathbf{\Sigma V}'^{\mathrm{H}}$ where $\mathbf{U}' = \mathbf{U}\mathrm{e}^{\mathrm{j}\theta}$, $\mathbf{V}' = \mathbf{V}\mathrm{e}^{\mathrm{j}\theta}$ and j is the imaginary unit is a SVD of $\mathbf{A}$ for any θ.