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Shape sensitivity analysis of large deformation frictional contact problems

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Abstract

Sensitivity analysis of large displacement multi-body two-dimensional contact problems with friction is developed in the paper. The incremental (path-dependent) sensitivity problem is derived by direct differentiation of the discretized equations governing the direct problem. In view of finite deformations, due attention is paid to spatial and nominal contact tractions and to proper formulation of the contact laws within the penalty approach. For these reasons an extended node-to-segment contact element is used to model the frictional contact interactions. As the finite elasto-plastic deformations of the contacting bodies are considered, the numerical procedures for computation of all the necessary characteristic formulae of the solid elements (for both the direct and the sensitivity problem) are automatically derived and generated using the symbolic algebra package *AceGen*. Numerical examples of shape and parameter sensitivity analysis illustrate the approach. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The sensitivity analysis (SA) deals with the prediction of structural response variation due to a given variation of the data describing the problem without re-computation of the perturbed problem. The sensitivity analysis is already a well established field of mechanics [1–4]. In the case of non-linear path-dependent problems, the general formulations are already developed both in a continuum [5,6] and in a discretized [7] format. However, the practical applications for problems with geometrical (finite deformations) and material (elasto-plasticity) non-linearities are still seldom, mostly due to severe complexity of the derivations involved. This is even more pronounced when frictional contact interactions are accounted for.

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The increasing interest in computational contact mechanics observed during the last decade and the active research in this field [8–11] lead to more and more robust finite element formulations so that many direct problems involving frictional contact may now efficiently be solved. As the SA of frictional contact problems is considered, several developments have recently been reported, however, in all cases only restricted classes of problems have been dealt with. The parameter (non-shape) SA is presented in [12] for frictional contact of elastic composite structures with a rigid plane. A continuum-based shape SA for large elasto-viscoplastic deformation problems with Coulomb friction is developed in [13,14]. The approach is restricted to contact with a rigid surface. In the direct problem the contact and friction conditions are enforced using an augmented Lagrangian technique, however, for the SA the regularization of the contact and friction conditions is performed by adopting oversized penalties. The shape variation of the rigid surface is excluded in [13] and the rigid tool shape variation is only considered in [14]. A continuum-based shape SA is also developed in [15]. Here the penalty method is used to enforce contact and friction conditions and again contact with a rigid surface is only considered. Note that all the above SA formulations are restricted to two-dimensional contact problems.

The flow approach assuming rigid-plastic material behaviour is a popular method of analysis of metal forming processes. The SA formulations for respective two-dimensional problems involving contact of the workpiece with a rigid tool are developed for the steady-state processes [16] and for the transient problems [17–19] in which the evolution of the configuration is obtained with an explicit time integration scheme.

In this paper we further develop the SA for two-dimensional frictional contact problems presented in [20]. The direct problem of contact of deformable bodies (multi-body contact) is formulated using linear node-to-segment contact elements [21] and the nominal penalty formulation [22] is consistently used in the present work. [1] The direct differentiation of governing equations following from the finite element discretization leads to the discrete sensitivity problem. As the frictional contact is a path-dependent phenomenon, the direct differentiation method (DDM) is chosen as it is superior to the adjoint system method for this class of problems [23]. The contact and friction conditions are enforced using the penalty method so that a pure displacement formulation is obtained and the SA approach of [7] can directly be applied. The shape and non-shape parameters are treated within the same unified framework and the sensitivity problem of large deformation multi-body frictional contact is obtained (not reported in the literature up to date). In the numerical examples SA is performed for shape variations leading to significant changes in the contact conditions. Also, cyclic loading leading to multiple contact closure–separation and forward–reverse sliding cycles is considered.

A computer symbolic algebra system allowing for symbolic derivation and automatic differentiation (AD) is used to numerically implement the sensitivity expressions for the continuum (solid elements with an elastic–plastic constitutive law). The system consists of two major components: the *Mathematica* [24] package *AceGen* that is used for the automatic derivation of formulae and code generation [25,26], and the *Mathematica* package *Computational Templates* with the prearranged modules for the creation of the finite element codes [27]. The *Computational Templates* package enables the generation of multi-language and multi-environment finite element codes from the same abstract symbolic description. More details about the formulation and the system are given in the Appendix A, however, in the present paper we mainly focus on the SA aspects of the frictional contact.

The paper is organized as follows. In Section 2 the general framework of DDM-based SA for discretized path-dependent problems [7] is briefly summarized. Next, in Section 3 the formulation of the direct problem of frictional contact is presented, followed by the respective sensitivity analysis formulation, Section 4. Finally, the approach and the accuracy of the SA is illustrated in Section 5 by the numerical examples.

[1] On the contrary, the spatial penalty formulation has been adopted in [20]. This formulation is, however, fundamentally less appropriate, as discussed in [22].