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SENSITIVITY ANALYSIS AS A METHOD OF ABSORBER TUNING FOR REDUCTION OF STEADY STATE RESPONSE OF LINEAR PARAMETRIC SYSTEMS

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The aim of this paper is to determine whether one dynamic absorber can reduce the amplitude of the steady state vibration of a parametric system for natural and parametric resonance frequencies simultaneously. The efficiency of both the conventional dynamic absorber and the parametric absorber is analyzed. The first order sensitivity analysis of parametric periodic systems in the time domain is applied to obtain logarithmic sensitivity functions in the frequency domain. The first order sensitivity logarithmic functions are used to tune the conventional absorber and the parametric absorber.

1. INTRODUCTION

A linear dynamical system subjected to combined parametric and forcing excitations of a periodic nature is governed by a system of inhomogeneous differential equations with periodic coefficients. A steady state periodic response of such a system was analyzed by Hsu and Cheng [1] and Klasztorny and Wójcicki [2]. The amplitudes of the steady state response of the system become unlimited on the stability boundaries of a homogeneous system corresponding to the non-homogeneous one. There are three sets of boundary curves separating stability regions from regions of instability. The first set (an even order one when the greatest multiplier is equal to 1 and a T-periodic solution exists) is characterized by the fact that near its boundary curves the steady state response amplitude of the system is amplified (unlimited) only when the period of the forcing excitation is an odd multiple of the period of the parametric excitation. For the second set (an odd order one when the greatest multiplier is equal to -1 and a $2T$-periodic solution exists) the steady state response amplitude of the structure is amplified (unlimited) only when the period of the forcing excitation is an even multiple of the period of the parametric excitation [1, 2]. The third set of boundary curves (the absolute value of the greatest multiplier is equal to 1 separates stability regions from regions of combined parametric instability. The steady state response amplitude of the system is not amplified (is always limited) near these boundaries.

The problem of optimal tuning of the absorber for an undamped single-degree-of-freedom system subjected to harmonic forcing excitation was formulated and solved by Den Hartog [3]; see also Harris [4]. The absorber's stiffness and damping are the parameters which change during the optimization procedure.

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The most effective reduction in the steady state response of the parametric system near the resonance areas is a more complicated problem, especially as there are no analytic solutions. The main difficulty is that the dynamic absorber must be tuned for more than one parametric resonance frequency. Hence, it is difficult to introduce a suitable optimization method. Optimal tuning for one frequency (force to parametrical excitation) ratio cannot be (usually is not) optimal for another one. Moreover, unlike any other kind of vibration, the resonance response amplitude in a linear parametric system may increase boundlessly in spite of the damping in the primary system [5]. Thus, damping must be involved to eliminate the unstable regions [5]. It therefore also becomes necessary to investigate the instability regions according to the Floquet theory [6]. This stability analysis is necessary as it makes little sense to talk about a steady state response of the system if the system is unstable [1, 2, 6]. Therefore, the optimization method ought to contain an effective stabilization method, for instance, similar to the one proposed by Seyranian *et al.* [7]. Thus, the stability of the system has to be checked continuously and an objective optimization function is difficult to define. Moreover, many parameters have to be changed. The interactive participation of a designer in the tuning procedure seems to be a better solution. A decision about which parameter and how much it should be changed to obtain better efficiency of the absorber is quite simple if the sensitivity function values are known.

When a harmonic force is the only one to excite the primary undamped system, an auxiliary mass damper in the resonance zone (close to the natural frequency of the primary system) is effective [3, 8]. In the present paper, the efficiency of a vibration absorber, when parametric excitation appears beside the forcing excitation, is analyzed. Moreover, a case of parametric excitation under a constant load is also examined [9, 10].

It is assumed that parametric instability resonance does not occur but excitation is very close to an instability region. Then the steady state response amplitude values may be very high [1, 2]. The most important question to examine is whether one dynamic absorber can reduce the amplitude of vibration of the primary system for both natural and parametric resonance frequencies. From this point of view, the application of a dynamic absorber to reduce the vibration of a single-degree-of-freedom parametric system is similar to the problem of a multi-degree-of-freedom system with many resonant peaks [8].

A numerical verification of the above question is the aim of this paper. The first order logarithmic sensitivity functions [11–13] are used to tune the dynamic absorber. The numerical calculations are aided by the *Mathematica* computer software [14].

2. STABILITY AND SENSITIVITY ANALYSIS

2.1. MATRIX EQUATION OF MOTION

The matrix equation of motion has the form

$$\mathbf{B}(t)\,\ddot{\mathbf{q}} + \mathbf{C}(t)\,\dot{\mathbf{q}} + \mathbf{K}(t)\,\mathbf{q} = \mathbf{f}(t), \tag{1}$$

where overdots refer to differentiation with respect to time t; $\mathbf{q}(t)$ is an n-dimensional vector of the generalized co-ordinates; $\mathbf{f}(t)$ is an n-dimensional force vector periodic in t with period T_f; and $\mathbf{B}$, $\mathbf{C}$, $\mathbf{K}$ are square n-dimensional real matrices of inertia, damping and stiffness respectively. Matrices $\mathbf{B}(t)$, $\mathbf{C}(t)$, $\mathbf{K}(t)$ are also periodic in t but with period T_0. Let $T_f = T_0 m_0/m_f$, where m_0 and m_f are assumed to be positive integers and m_0/m_f is an irreducible rotational number. Thus, the forcing period of $\mathbf{f}(t)$ is commensurate with the period of coefficient matrices $\mathbf{B}$, $\mathbf{C}$, $\mathbf{K}$. Let T_c be a common period between parametric and