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FUZZY SENSITIVITY ANALYSIS FOR THE IDENTIFICATION OF MATERIAL PROPERTIES OF ORTHOTROPIC PLATES FROM NATURAL FREQUENCIES

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To enhance the identification of material properties of orthotropic plates, a special method of fuzzy sensitivity analysis is presented which allows the determination of the dependencies between the material properties and the natural frequencies of each mode. The procedure is based on the classical thin-plate theory in conjunction with a recently developed method for calculating the natural frequencies of completely free orthotropic plates using the exact series solution. After presenting the fundamentals of fuzzy sensitivity analysis which include the numerical implementation of uncertain parameters as well as the simulation and the analysis of the model, the results of the proposed method applied to vibrations of an orthotropic plate are given.

1. INTRODUCTION

For the identification of stiffness parameter of orthotropic plates, Caldersmith [1] suggested a method using the concept of sinusoidal equivalent length and applied it to wooden plates. However, in the paper he limited his work to identifying the stiffness parameters only. An extension of this work was done in reference [2] where the elastic constants were determined using the assumption that the product of Poisson's ratios $\nu_x \nu_y$ in the orthotropic plate is the same as the product of Poisson's ratio ν^2 in the isotropic case. The disadvantage of this method is that one has first to identify the Poisson's ratio of a corresponding isotropic plate. Ayorinde and Yu [3] suggested to use the diagonal modes for identifying the material properties of plates. However, this method is only advisable for isotropic plates because otherwise one needs an orthotropic plate with unknown specific dimensions in order to obtain diagonal modes. De Wilde *et al*. [4, 5] used an optimisation method to solve for the elastic constants. He used an overdetermined set of equations in order to obtain the six elastic rigidities of rectangular anisotropic plates. Frederiksen [6–8] tested different ceramics composite and fibre-reinforced epoxy panels for which both elastic constants governing the classical thin-plate theory and constants associated with the thick-plate theory were estimated. Larsson [9] matched the results from experimental modal testing with theoretical modal analysis calculations for a set of plate bending modes and one in-plane mode of the compression type. The elastic constants are estimated by minimising the relative errors between corresponding experimentally and theoretically determined natural frequencies.

Common techniques of model updating usually suffer from the need of verification, validation, usability and falsification of the model [10–13]. Moreover, identification of the material properties from experimental data shows a serious problem which mainly consists

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in the acquisition of significant data. As can be shown, one can always get some values determined for the material properties on the basis of experimental data, but there is still the question whether these results are reasonable or not. It turns out that to guarantee the identification of reasonable results, the experimental data must be acquired for those eigenfrequencies and vibration modes only where a certain material parameter shows maximum influence on the vibration behaviour. To practically determine the degrees of influence of the material parameters, a special method of sensibility analysis based on fuzzy arithmetic has been introduced by Hanss [14] as a part of the so-called transformation method. The objective of this paper is to analyse the vibration behaviour of a rectangular orthotropic Kirchhoff plate with free boundaries with respect to the influence of its material properties, and finally, to give recommendations about which vibration modes are to be considered for data acquisition.

2. CALCULATION OF THE NATURAL FREQUENCIES OF ORTHOTROPIC PLATES

Vibrations of plates have been frequently studied. Unfortunately, there exists no closed-form solution for the case of a rectangular Kirchhoff plate with free boundaries, but several approximate methods have been proposed. Warburton [15] used characteristic beam vibration functions in Rayleigh's method [16] to obtain a useful, simple approximate expression for the natural frequencies of vibrations of thin, isotropic plates. His work was extended by Hearmon [17] and applied to special orthotropic plates and by Dickinson [18] to include the effect of uniform in-plane loads. Warburton's expression, and its generalisations, together with a table permit the straightforward calculation of the natural frequencies of plates having any combination of free clamped or simply supported edges. However, if there exist one or more free edges, then the accuracy of the frequencies can be significantly diminished. Kim and Dickinson [19] provided an improved approximate expression where they use Rayleigh's method but in connection with the minimum potential energy theorem.

Leissa [20] presented comprehensive and accurate analytical results for the free vibration of rectangular plates. He applied the Ritz method [21] and compared the results to the method of Warburton [15]. But, his work is limited to isotropic plates. Leissa's work was extended by Deobald and Gibson [22], who applied the Rayleigh–Ritz method to orthotropic plates as well. Wang and Lin [23] presented a systematic analysis for solving boundary value problems in structural mechanics where a weighted residual form of the differential equations is used with sinusoidal weighting functions. Recently, this appproach has been extended to calculating the eigenfrequencies and eigenmodes of an orthotropic plate with completely free boundary using an exact series solution by Hurlebaus *et al.* [24]. This section summarises the basic ideas of the previously mentioned method in order to give an understanding on how the fuzzy sensitivity analysis is based on.

The partial differential equation governing the free transverse vibration of a symmetrically laminated thin plate (by neglecting shear effect and rotatory inertia) at equilibrium in the $x-y$ plane may be written in terms of the moments as

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \rho h \frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where x and y are the orthogonal plane coordinates, and $w = w(x, y, t)$ is the plate deflection. Separation of variables with $w(x, y, t) = W(x, y)\exp(i\omega t)$, weighting with $\cos\alpha_m x \cos\gamma_n y$, where $\alpha_m = m\pi/a$ and $\gamma_n = n\pi/b$, and integration with respect to the plate area $a \times b$ leads to an integral equation, where the first three terms can be integrated by