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Sensitivity Analysis Framework for General Quasi-Variational Inclusions

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Abstract—It is well known that the implicit resolvent equations are equivalent to the quasi-variational inclusions. We use this alternative equivalent formulation to study the sensitivity of the quasi-variational inclusions without assuming the differentiability of the given data. Since the quasi-variational inclusions include classical variational inequalities, quasi (mixed) variational inequalities, and complementarity problems as special cases, results obtained in this paper continue to hold for these problems. In fact, our results can be considered as a significant extension of previously known results.

Keywords—Variational inclusions, Sensitivity analysis, Resolvent equations, Parametric equations.

1. INTRODUCTION

Quasi-variational inclusions are being used as mathematical programming models to study a large number of equilibrium problems arising in finance, economics, transportation, optimization, operations research, and engineering sciences. The behavior of such equilibrium solutions as a result of changes in the problem data is always of concern. In this paper, we study the sensitivity analysis of quasi-variational inclusions, that is, examining how solutions of such problems change when the data of the problems are changed. We remark that sensitivity analysis is important for several reasons. First, since estimating problem data often introduces measurement errors, sensitivity analysis helps in identifying sensitive parameters that should be obtained with relatively high accuracy. Second, sensitivity analysis may help to predict the future changes of the equilibrium as a result of changes in the governing systems. Third, sensitivity analysis provides useful information for designing or planning various equilibrium systems. Furthermore, from mathematical and engineering points of view, sensitivity analysis can provide new insight regarding problems being studied and can stimulate new ideas for problem solving. Over the last decade, there has been increasing interest in studying the sensitivity analysis of variational inequalities and variational inclusions. Sensitivity analysis for variational inclusions and inequalities has been studied by many authors including Tobin [1], Kyparisis [2,3], Dafermos [4], Qiu and Magnanti [5], Yen [6], Noor [7,8], Moudafi and Noor [9], Noor and Noor [10], and Liu [11] using

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quite different techniques. The techniques suggested so far vary with the problem being studied. Dafermos [4] used the fixed-point formulation to consider the sensitivity analysis of the classical variational inequalities. This technique has been modified and extended by many authors for studying the sensitivity analysis of other classes of variational inequalities and variational inclusions; see [6–9,12,13] and the references therein. In this paper, we extend this technique to study the sensitivity analysis of general quasi-variational inclusions. We first establish the equivalence between the general quasi-variational inclusions and implicit resolvent equations by using the resolvent operator method. This fixed-point formulation is obtained by a suitable and appropriate rearrangement of the implicit resolvent equations. We would like to point out that the resolvent equations technique is quite general, unified, flexible, and provides us with a new approach to study the sensitivity analysis of variational inclusions and related optimization problems. We use this equivalence to develop sensitivity analysis for the general quasi-variational inclusions without assuming the differentiability of the given data. Our results can be considered as a significant extension of the results of Dafermos [4], Moudafi and Noor [9], Noor and Noor [10], and others in this area.

2. PRELIMINARIES

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle .,. \rangle$ and $\|.\|$, respectively. Let $N(.,.), A(.,.) : H \times H \to H$ be two nonlinear operators. We consider the problem of finding $u \in H$ such that

$$0 \in N(u,u) + A(g(u),u). \tag{2.1}$$

Inclusion of type (2.1) is called the general mixed quasi-variational inclusion, which has many important and useful applications in pure and applied sciences. We note that if $A(.,.) = \partial\varphi(.,.)$, where $\partial\varphi(.,.)$ is the subdifferential of a proper, convex, and lower-semicontinuous function $\varphi(.,.) : H \times H \to R \cup \{+\infty\}$ with respect to the first argument, then problem (2.1) is equivalent to finding $u \in H$ such that

$$0 \in N(u,u) + \partial\varphi(g(u),u),$$

or equivalently

$$\langle N(u,u), g(v) - g(u)\rangle + \varphi(g(v),g(u)) - \varphi(g(u),g(u)) \geq 0, \qquad \text{for all } v \in H,$$

which is known as the general mixed quasi-variational inequality. For the applications and numerical methods of variational inclusions and variational inequalities, see [1–20].

We note that if $\varphi(.,u) \equiv \varphi(u)$ is the indicator function of a closed convex set K in H, then problem (2.1) is equivalent to finding $u \in H$, $g(u) \in K$ such that

$$\langle N(u,u), g(v) - g(u)\rangle \geq 0, \qquad \text{for all } g(v) \in K, \tag{2.2}$$

which is called the general variational inequality. If $N(u,u) = Tu + V(u)$, where $T, V : H \longrightarrow H$ are single valued operators, then problem (2.2) is called the strongly nonlinear variational inequality, studied and considered by Noor [19]. For recent applications, numerical methods, sensitivity analysis, and physical formulations, see [1–20] and the references therein.

We recall that if T is a maximal monotone operator, then the resolvent operator J_T associated with T is defined by

$$J_T(u) = (I + \rho T)^{-1}(u), \qquad \text{for all } u \in H,$$

where $\rho > 0$ is a constant and I is the identity operator. The resolvent operator J_T is a single-valued operator and is nonexpansive.

REMARK 2.1. Since the operator $A(.,.)$ is a maximal monotone operator with respect to the first argument, we define

$$J_{A(u)}u = (I + \rho A(u))^{-1}u, \qquad \text{for all } u \in H,$$

the implicit resolvent operator associated with $A(.,u) \equiv A(u)$.