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An implementation of the stiffness derivative method as a discrete analytical sensitivity analysis and its application to mixed mode in LEFM

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Abstract

In this work, an improvement in the stiffness derivative method based on a shape design sensitivity analysis is proposed, so that the error inherent in the finite difference procedure is avoided. For a global estimation of G from a given finite element solution, this approach is shown to be equivalent to the well-known J-integral when the latter is numerically implemented through its equivalent domain integral. However, it is verified that its direct application to 2D mixed mode problems of linear elastic fracture mechanics through the field decomposition technique yields estimates for G_{I} and G_{II} which are in general more accurate for the proposed method. The importance of the velocity field is also remarked and some suggestions for its choice are given.

Keywords: Finite element analysis; Stiffness derivative method; Equivalent domain integral method; Sensitivity analysis; Mixed mode fracture

1. Introduction

A number of numerical techniques exist which permit to obtain the SIFs or equivalently the SERR in linear elastic fracture mechanics (LEFM) from a FE solution. Among them, the so-called *indirect methods* are considered as the most accurate and efficient [1–4]. These methods are based on an energetic approach, and for them, the SERR G is the characterizing parameter. By contrast, *direct methods* yield an estimate of the SIF K through a local approach, without calculating the corresponding value of G. Therefore they need a refined mesh around the crack tip and the use of special crack tip elements. Generally speaking, all these methods are post-processing techniques which are applied after performing a numerical analysis, such a FE analysis (FEA).

Some of the indirect methods most commonly used in the literature are certain contour integrals, such as the J integral [5], the stiffness derivative method, proposed independently by Parks [2] and Hellen [3], the

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Nomenclature

a	crack length (design variable)
$\mathbf{B}$	element strain field–nodal displacement relationship matrix
$\mathbf{D}$	stress–strain constitutive matrix
δ_{ij}	Kronecker's delta
δa	virtual crack extension
Δa	finite difference increment for a crack extension
E, E'	elasticity modulus and generalized elasticity modulus
$\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon}^{\mathrm{I}}$, $\boldsymbol{\varepsilon}^{\mathrm{II}}$	strain field and its decomposed parts for mode I and II (vector notation)
$\varepsilon^{\mathrm{I}}_{ij}$, $\varepsilon^{\mathrm{II}}_{ij}$	decomposed parts for the strain field (indicial notation)
$\boldsymbol{\varepsilon}_{\mathrm{fe}}$	strain field within an element derived from the finite element (FE) nodal solution
G, G_{I}, G_{II}, G_{III}	strain energy release rate (SERR). Global, mode I, mode II and mode III values as obtained through the stiffness derivative method
Γ_0, Γ_1	contours around the crack tip
$\mathbf{I}$	identity matrix
J, J_{I}, J_{II}, J_{III}	SERR. Global, mode I, mode II and mode III values as obtained through the equivalent domain integral (EDI) method
$\mathbf{J}$	Jacobian matrix of the isoparametric coordinate transformation
$\overline{\mathbf{K}}_{\mathrm{fe}}, \mathbf{K}^{\mathrm{e}}$	global and element stiffness matrices for a given FE discretization
$\mathrm{d}\mathbf{K}^e/\mathrm{d}a$	sensitivity of the element stiffness matrix
K_{I}, K_{II}, K_{III}	stress intensity factor (SIF) (mode I, mode II and mode III)
N_e	number of elements of a FE discretization
N_{ne}	number of nodes per element
N_n	shape function defined at node n
$N_{n,x}, N_{n,y}, N_{n,z}$	partial derivatives of the shape function defined at node n with respect to global coordinates
$N_{n,\xi}, N_{n,\eta}, N_{n,\zeta}$	partial derivatives of the shape function defined at node n with respect to the reference element coordinates
ν	Poisson's ratio
Π	total potential energy
r, θ	polar coordinates with origin at the crack tip
$\boldsymbol{\sigma}$, $\boldsymbol{\sigma}^{\mathrm{I}}$, $\boldsymbol{\sigma}^{\mathrm{II}}$	stress field and its decomposed parts for mode I and II (vector notation)
σ_{ij}, σ'_{ij}	stress field at a point P and at its symmetric point P' with respect to the crack plane (indicial notation)
σ^{I}_{ij}, $\sigma^{\mathrm{II}}_{ij}$	decomposed parts for the stress field (indicial notation)
$\boldsymbol{\sigma}_{\mathrm{fe}}$, $\sigma_{ij_{\mathrm{fe}}}$	stress field within an element derived from the FE nodal solution and its components
$\mathbf{u}$, $\mathbf{u}^{\mathrm{I}}$, $\mathbf{u}^{\mathrm{II}}$	displacement field and its decomposed parts for mode I and II (vector notation)
u_i, u'_i	displacement field at a point P and at its symmetric point P' with respect to the crack plane (indicial notation)
u^{I}_i, u^{II}_i	decomposed parts for the displacement field (indicial notation)
$\bar{\mathbf{u}}_{\mathrm{fe}}$, $\mathbf{u}^e$, $\mathbf{u}^{\mathrm{I},e}$, $\mathbf{u}^{\mathrm{II},\mathrm{e}}$	global and element vector of the nodal solution for displacements in a FEA and decomposed parts for the latter into mode I and mode II components
$\mathbf{u}_{\mathrm{fe}}$, $u_{i_{\mathrm{fe}}}$	displacement field within an element derived from the FE nodal solution and its components
U	strain energy
$\mathbf{v} \equiv \mathbf{q}$	velocity field (called **q**-function in the EDI method)